NZ IMAges

ISSUE FIVE



# Modelling and heart patients

**Staff at the Cardio-thoracic and Vascular Intensive Care Unit (ICU) of Auckland City Hospital have found a mathematical model of the unit's operation valuable in improving its efficiency. Jenny Rankine reports.**

Unit clinical director Dr Andrew McKee says juggling staffing, theatre availability and beds is complex; "it's hard to match the resources to the demand".

A chance conference meeting in 2006 started a collaboration between statistician Ilze Ziedins, pictured, BSc (Hons) and Masters student William Chen, and unit staff on a queuing model that could simulate the effects of operational changes on patient numbers. Associate Professor Ross Ihaka co-supervised Chen's Master's thesis, and advised on constructing the simulation and other aspects of the project.

At the time, the unit admitted around 22 patients a week, some for elective surgery and others with acute problems needing intensive care. "The bottleneck was the intensive care unit," says Ziedins. "Around half the patients stay for a day or less, some stay for much longer. We modelled the flow of scheduled elective and other patients into the unit, with random acute arrivals and lengths of stay, simulating 24 hours and seven days a week."

The model gradually became more complicated, taking into account the cluster of arrivals around midday and after 4pm after surgery, variations by day of the week and different kinds of patients. "Since arrival rates change over the duration of a patient's stay, traditional queueing models are not helpful, and new analytical models will need to be developed," says Ziedins.

Chen wrote the simulation programme from scratch using the statistical software R (see IMAges 3). Each simulation run was for a year of the model's operation, and this was repeated several times to obtain confidence intervals for measures such as the average number of cancellations. The initial aims were to reduce waiting times, and cancellations of elective surgery due to the arrival of people with acute problems. "The model demonstrated that we needed more staffed ICU beds to match operating theatre capacity," says McKee. "We had an average of nine and needed 12 to manage our expected number of patients."

Having an external analysis independent of clinical pressures was a powerful argument for more staff, he says. The unit now has a higher allocated staffing level, although the international shortage of clinical staff has meant not all the positions have been filled.

The aim then shifted to matching the nursing roster with the patient load. "We're working on that now using a stochastic optimisation model," says Ziedins. As rosters are done three months in advance, the evaluation cannot start until the current roster ends in mid-November. "We think improvements can be made; they may be able to treat one or two more patients a week, which is substantial over a year."

"The whole unit has been very interested," says Ziedins. "Up to 20 people have turned up for

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## Welcome

We hope you enjoy this, our fifth issue of IMAges, which contains a range of items about the work and interests of the New Zealand mathematical community, plus an item on the mathematics of origami as well as photos of a recent event where schoolchildren were invited to participate in the creation of a large Penrose Tiling. Read about this and more inside.

**Marston Conder and Vaughan Jones**
**Co-Directors**

## INSIDE



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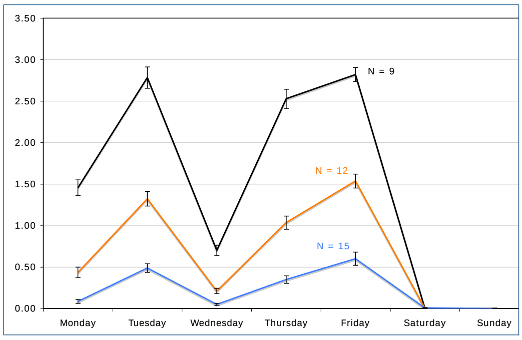
◀ I presentations; the input from them is wonderful."

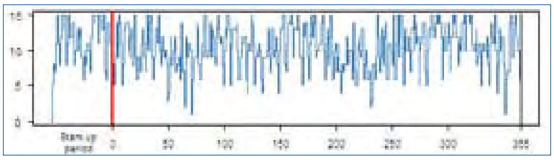
"We can use the model to analyse our patterns of work," says McKee. "For example, we can see if it would make any difference to patient throughput if we could discharge all the patients back to the ward an hour earlier after surgery."

Queuing theory is often used to analyse phone, internet and road networks, as well as customer services such as banking. "Once you think about life, almost everything starts to look like a queue," says Ziedins. It's the randomness that's important. Reducing variability can have a marked effect on how systems perform. For example, lights on Auckland motorway on ramps reduce clumping and make traffic flow less congested."

"I find this project so rewarding because it has been an opportunity to make a difference - some people might receive treatment earlier as a result," she says.




**Above: The average number of ICU cancellations per day, with different numbers of staffed beds. This plot was produced in the project's first year, when the model assumed a constant number of staffed beds during the week and not all patients were included. Only one operation session is scheduled on Wednesday, leaving time for meetings and training.**
**Far left: William Chen. Left: The number out of 15 beds occupied overnight for a single simulation run over a year.**



$$e^{-\lambda/\mu}\frac{(\lambda/\mu)^n}{n!}$$

"In how many other disciplines is there anything that parallels the statement that Gauss's formula for triangular numbers will ***never*** fail to give the right answer?"

*Marcus de Sautoy, Music of the Primes.*

## NOTABLE MATHS PROBLEMS

### COLLATZ CONJECTURE

**Take any positive integer; if even, divide by two; if odd, triple it and add one. Using each result as the input for the next step, no matter what number starts the sequence, the end result is 1.**

**Simply**: That all paths in a certain kind of number sequence eventually lead to 1.

**Also known as**: The $3n + 1$ conjecture, the Ulam conjecture (after Polish mathematician Stanislaw Ulam), the Syracuse problem or HOTPO (Half Or Triple Plus One) in computer programming.

**Discipline**: Number theory.

**Originator**: German mathematician Lothar Collatz, 1937; he made very little progress and published a history of its origin much later.

**Incentive**: $US50 by H Coxeter, 1970; then $US500 by Hungarian mathematician Paul Erdős; £1,000 by B Thwaites, 1996; solving a problem with a tantalizingly elementary form that has eluded top mathematicians.

**Examples**: If $n = 6$, the number sequence is 6, 3, 10, 5, 16, 8, 4, 2, 1. If $n = 27$, the sequence takes 111 steps, climbing to over 9,000 before descending to 1.

**Explorations**: Many attempts have been made to settle the conjecture using technologies from number theory, dynamical systems and Markov chains. USA mathematician Jeffrey Lagarias proved in 1985 that the problem has no nontrivial cycles of length <275,000. Another approach took the opposite direction; instead of proving that all natural numbers eventually lead to 1, it proved that 1 leads to all natural numbers. USA mathematician John Conway proved in 1972 that Collatz-type problems can be formally undecidable.

**State of play**: Although the conjecture has not been proved, most mathematicians who have worked on the problem believe it is true. The conjecture has been checked by computer for all starting values up to $10 \times 2^{58}$. However, some important conjectures have been found to be false only with very large counterexamples.

Each odd number in Collatz sequences is on average ¾ of the previous one, leading to an argument that every Collatz sequence should decrease in the long run. This is also not a proof because it pretends that the sequences are assembled from uncorrelated probabilistic events.

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