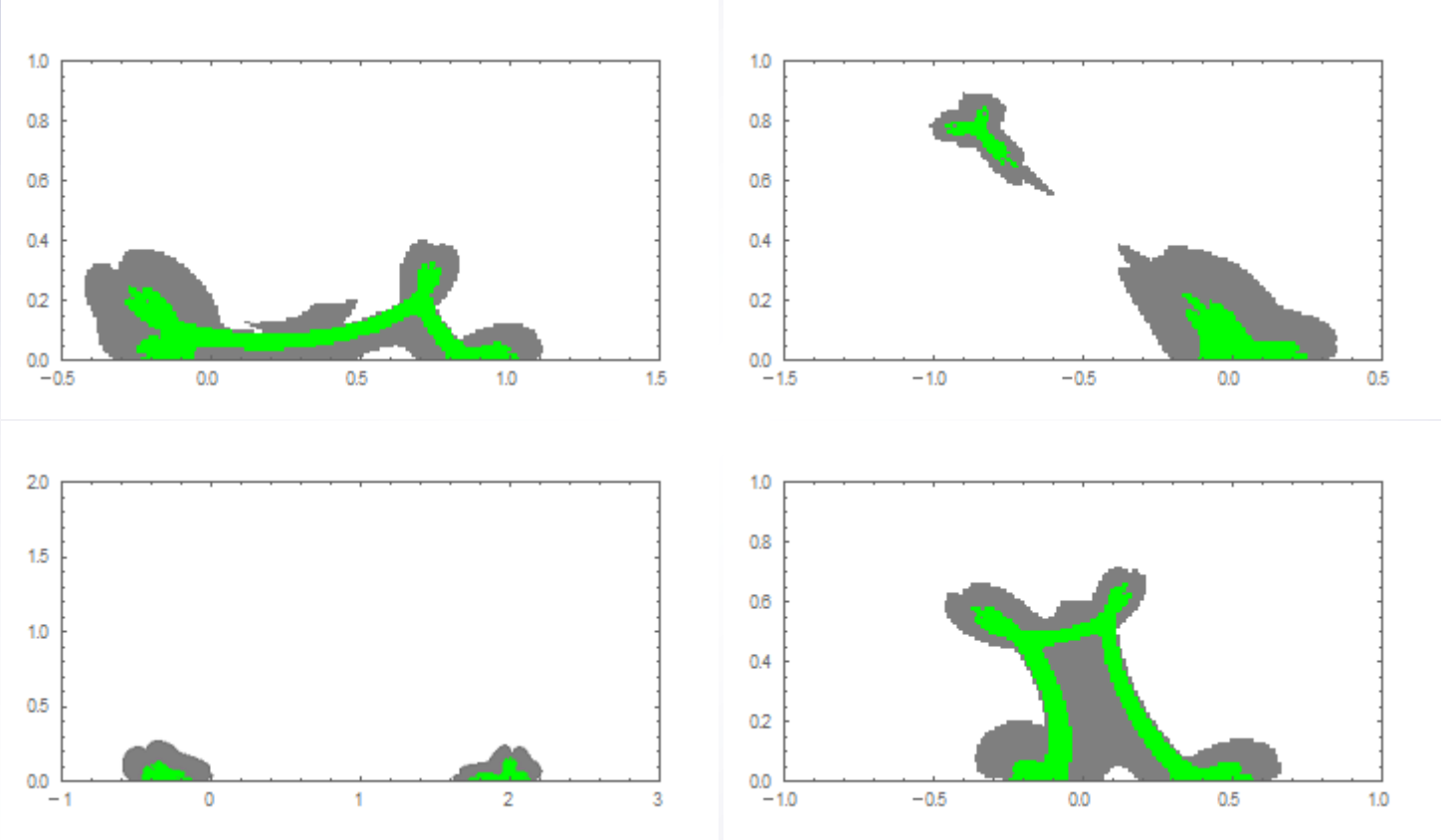
VBBinaryLensing **2.0**

A fast and robust code for light curve computation



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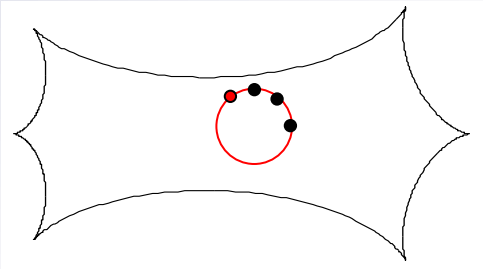
VBBinaryLensing

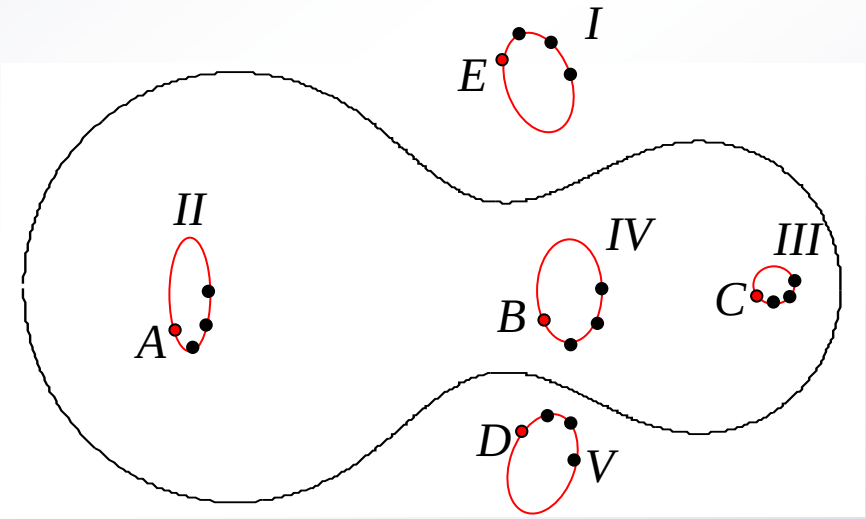
- **VBBinaryLensing** is a code for the calculation of **microlensing light curves**.
  - Point-Source Point-Lens
  - Extended Source Point-Lens
  - Binary Source Point-Lens
  - **Extended Source Binary Lens**
- **Higher order effects** implemented:
  - Linear limb darkening
  - Annual and space parallax
  - Circular orbital motion
- C++ library, importable in Python, source **code public**.
- Tested on Windows, Linux, Mac OS.

- Essential core of **RTModel** (http://www.fisica.unisa.it/GravitationAstrophysics/RTModel.htm).
- Incorporated in several high-level analysis frameworks:
  - **pyLIMA** *(Bachelet et al. 2017)*
  - **MulensModel** *(Poleski & Yee 2018)*
  - **MuLAN** *(Cassan & Ranc 2017)*

Magnification by Contour Integration

- VBBinaryLensing uses the **contour integration** method *(Schramm & Kayser 1987; Dominik 1993; Dominik 1995; Gould & Gaucherel 1997; Dominik 1998; Dong et al. 2006; Dominik 2007)*.





- **Parabolic correction** greatly increases the accuracy.
- **Error control** is performed through several error estimators.
- **Sampling** is **optimized** where/when really needed to match the accuracy goal.
- **Limb darkening** accounted by repeating the calculation on multiple annuli. *(V. Bozza, MNRAS 408, 2188 (2010))*

VBBinaryLensing 2.0: what's new?

- The **root solving routine** is now Skowron & Gould.
- **Relative precision goal** and/or absolute accuracy goal.
- **Extended source** calculation performed only **when** really **needed**
- Extended source-point lens calculation performed using **pre-calculated tables**.
- Light curve calculations on **large datasets** with a single call.

Skowron & Gould algorithm

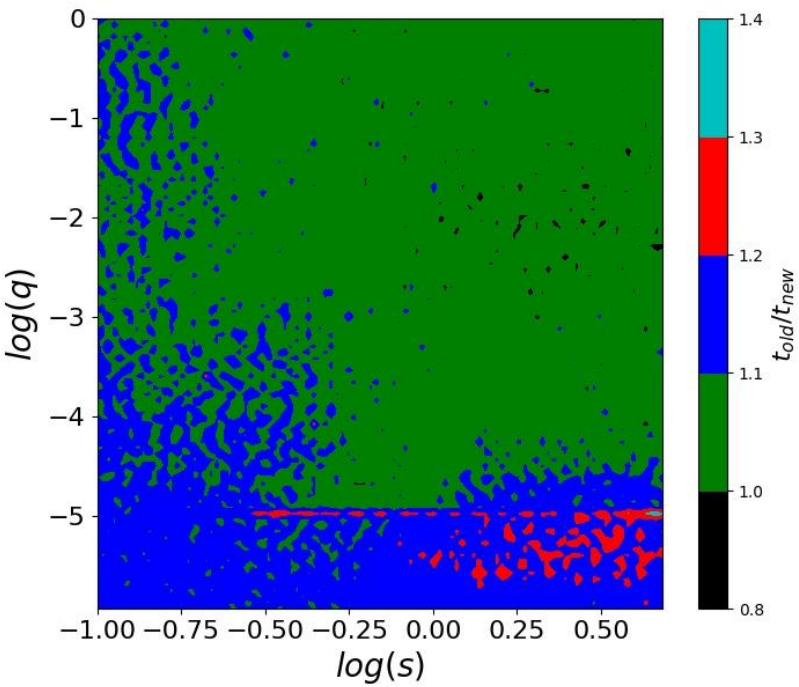
- The binary lens equation is equivalent to a fifth degree complex polynomial.

Two ways for **finding roots** *(Skowron & Gould arXiv:1203.1034)*

$$z_{k+1} = z_k - \frac{1}{G} \qquad G = \frac{p'(z_k)}{p(z_k)}$$

$$H = G^2 - \frac{p''(z_k)}{p(z_k)} \qquad z_{k+1} = z_k - \frac{n}{G \pm \sqrt{(n-1)(nH - G^2)}}$$

- Use **Newton**'s method when close enough to the goal.
- Use **Laguerre**'s method when still far away.



- **Speed-up** of order 1.1-1.2 with respect to `zroots` (Numerical Recipes).
- **More robust** for extreme parameters.
- The original Fortran code has been translated to C++ by **Tyler Heintz and Ava Hoag** (Westminster College) during a summer studentship in Salerno.

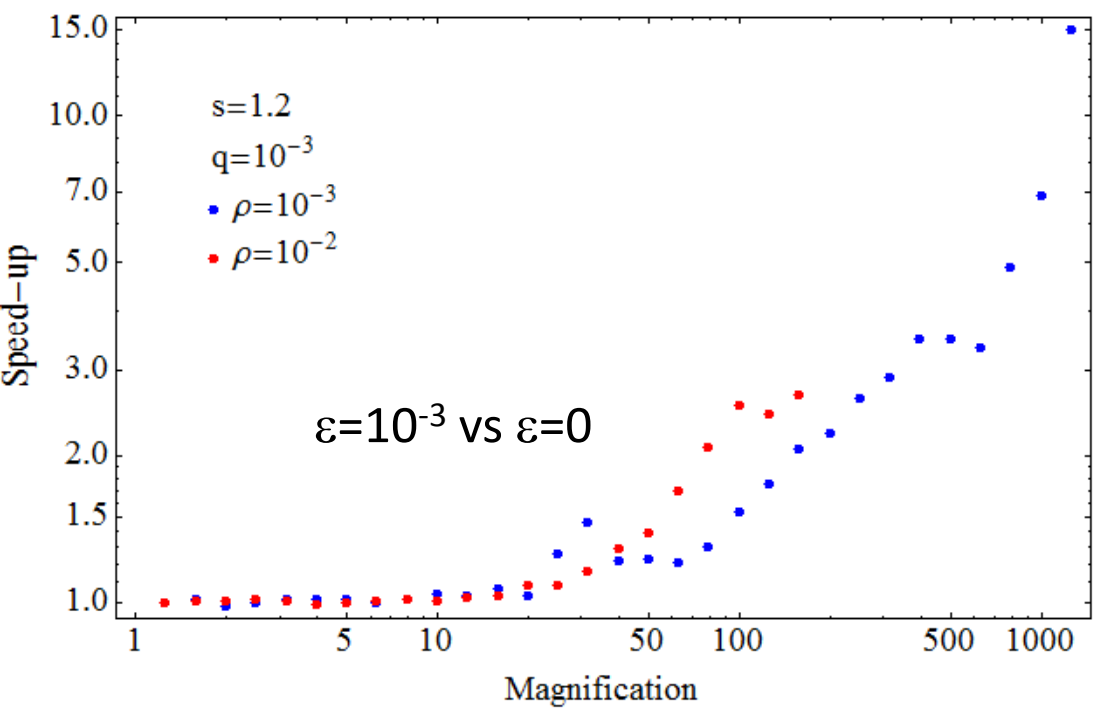
Precision vs Accuracy

- If the **accuracy goal** is δ, the calculation stops when the absolute error on the magnification is

$$\delta\mu < \delta$$

- If the **precision goal** is ε, the calculation stops when the relative error is

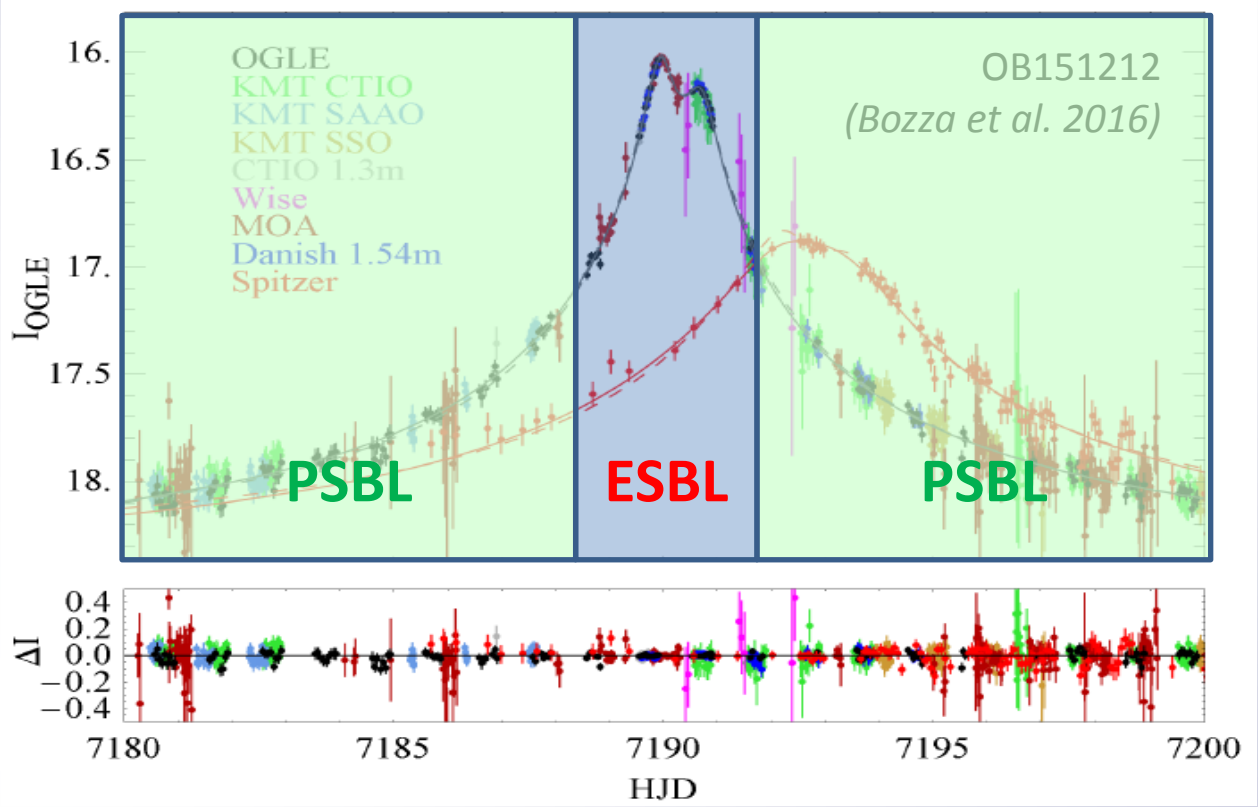
$$\frac{\delta\mu}{\mu} < \varepsilon$$



- At **high-magnification**, a millimag precision is sufficient.
- The calculation stops earlier and we get a significant **speed-up**.

PSBL vs ESBL

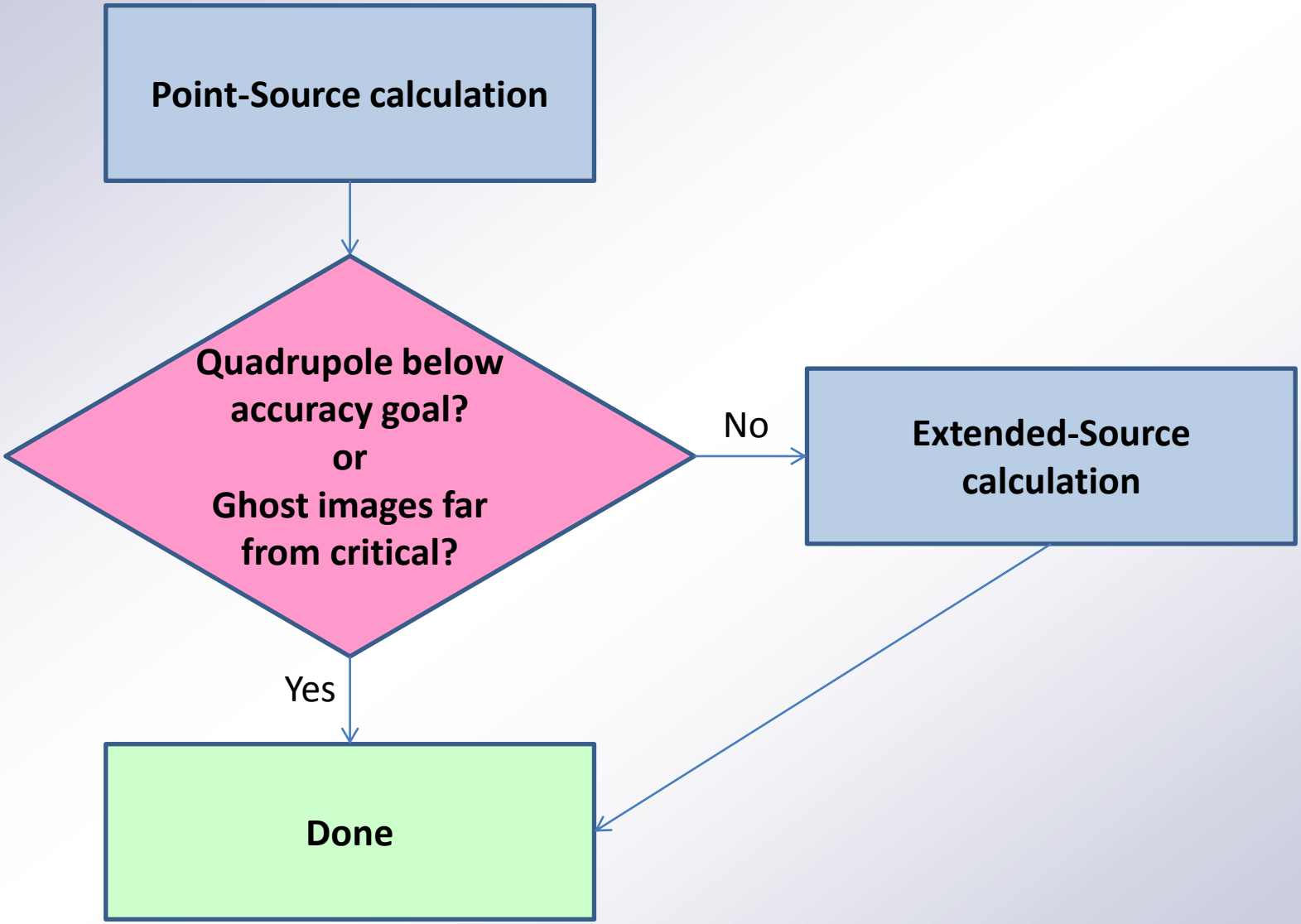
- **Point-Source approximation** works for most points in a microlensing light curve.
- Only regions around the caustics require the full **Extended-Source calculation**.
- In general, when modeling a specific event, the modeler defines **time windows** in which different codes are used.



- This approach is **unpractical** if we aim at modeling thousands of events automatically.

Tracking finite-source effects

- Pre-defined time windows are replaced by dynamical evaluation of specific metrics:



Finite-Source Metrics

- For each image, we can evaluate the **quadrupole** correction *(Pejcha & Heyrovsky 2009, Gould 2008, Cassan 2017)*

$$Q = 2\left|\frac{\mathrm{Re}\left[3\bar{f}^3 f'^2 - \left(3 - 3J + J^2/2\right)\left|f'\right|^2 + J\bar{f}^2 f''\right]}{J^5}\right|$$

$$f(z) = \sum_i \frac{m_i}{(z - z_i)^2}$$

$$Q_2 = \left|\frac{\mathrm{Im}\left[3\bar{f}^3 f'^2\right]}{J^5}\right|$$

Additional term for better protection around cusps

- **Ghost images** provide us with an approximate distance to caustics:

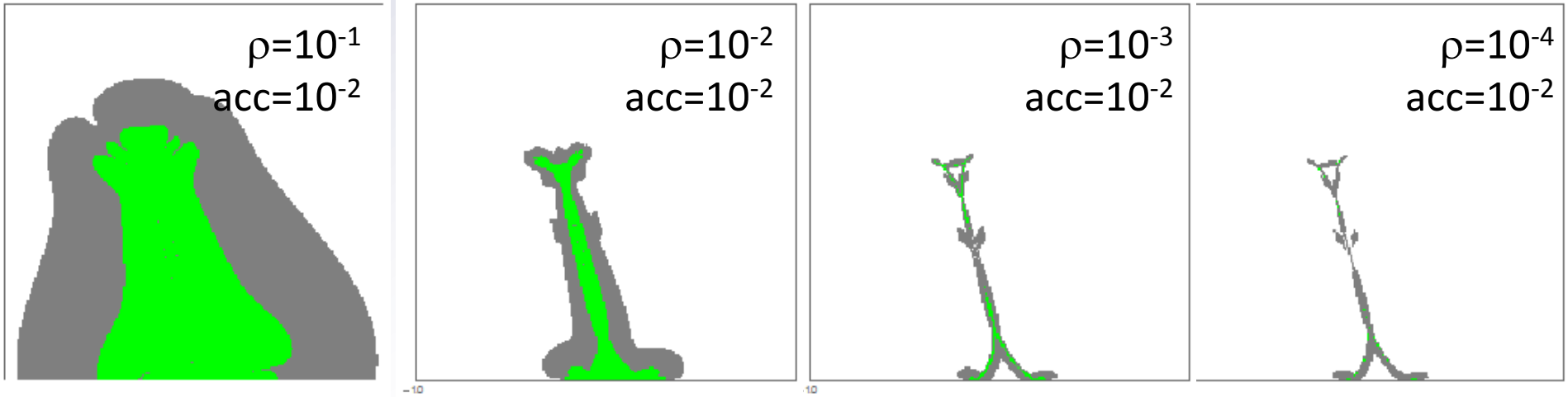
$$d_G = \left|\frac{J}{\nabla_\zeta J}\right|$$

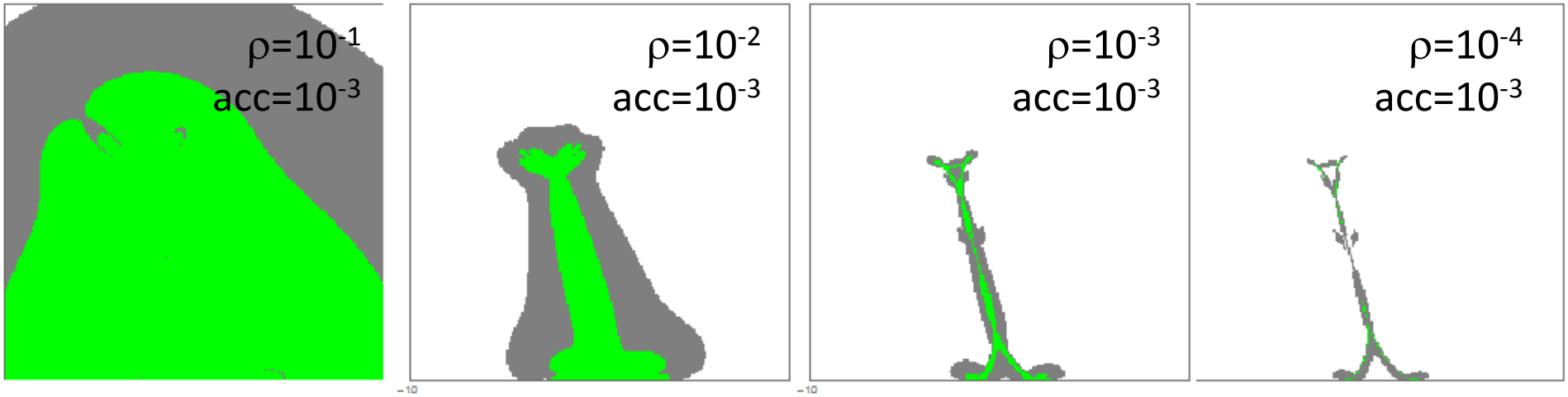
- Point-Source approximation is sufficient if

$$\boxed{\left(Q + Q_2\right)\rho^2 < accuracy \quad \mathrm{AND} \quad d_G > \rho}$$

Safety maps

- The safety criterion $\boxed{(Q+Q_2)\rho^2 < accuracy \quad \text{AND} \quad d_G > \rho}$ tells us when to go to full Extended-Source calculation.





Example of use

```
#include <stdio.h>
#include "VBBinaryLensingLibrary.h"

int main()
{
   VBBinaryLensing VBBL;

   double Mag,s,q,y1,y2,rho,a1,accuracy;
   s=0.8; //separation
   q=0.1; // mass ratio
   y1=0.01; // source position
   y2=0.3;
   rho=0.01; // source radius

   VBBL.a1=0.51; // Linear limb darkening coefficient
   VBBL.Tol=1.e-2; // Accuracy goal
   Mag=VBBL.BinaryMag2(s,q,y1,y2,rho);

   printf("Magnification = %lf\n",Mag);

   return 0;
}
```

Extended-Source-Point-Lens

- **Extended-Source-Point-Lens** has two regimes:

$$\mu = \begin{cases} \dfrac{u^2+2}{u\sqrt{u^2+4}} f_{out}\left(\frac{\rho}{u}, \rho\right) & \text{if} \quad u > \rho \\ \sqrt{1+4/\rho^2}\, f_{in}\left(\frac{u}{\rho}, \rho\right) & \text{if} \quad u < \rho \end{cases}$$

- Rather than calculating elliptic integrals, we evaluate the correction factors f_{in} and f_{out} by **pre-calculated tables**.
- This is much faster (and accurate) than any other methods.

- **Limb darkening** is obtained by dividing the source in annuli as in contour integration.

Outlook

- **VBBinaryLensing 2.0** achieves a significant speed-up by introducing new **decision trees** at several levels:
  - Skowron & Gould algorithm in root solving
  - Relative precision goal (great speed-up at high-mag)
  - Finite-Source used only when needed (great speed-up far from caustics).
- Code will still highly benefit from **continuous feedback** from the community.
- Extensive use within high-level analysis environments.
- Next step: **multiple lensing** with contour integration.

http://www.fisica.unisa.it/GravitationAstrophysics/VBBinaryLensing.htm.