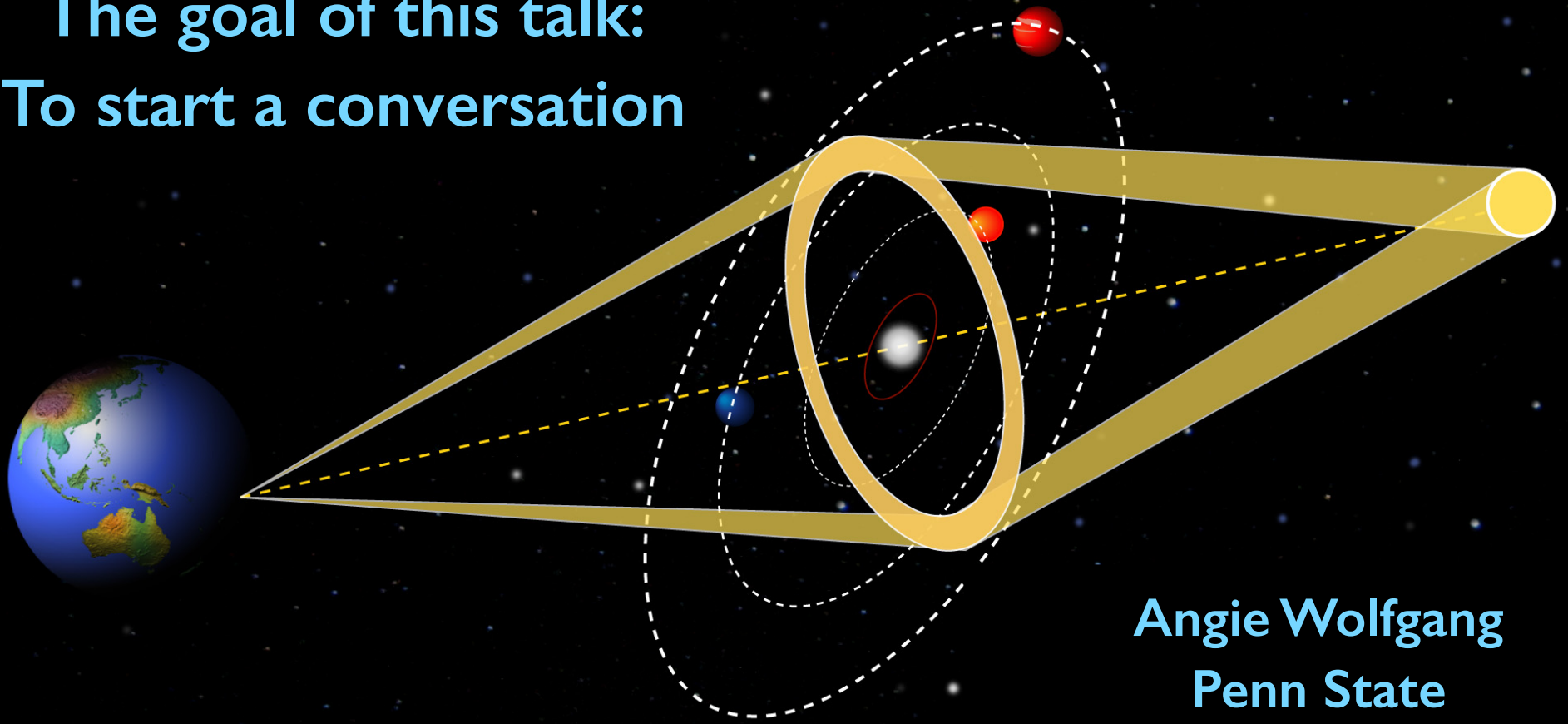


# Statistical Opportunities in Microlensing

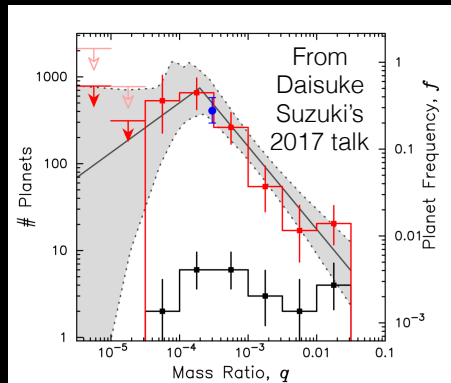
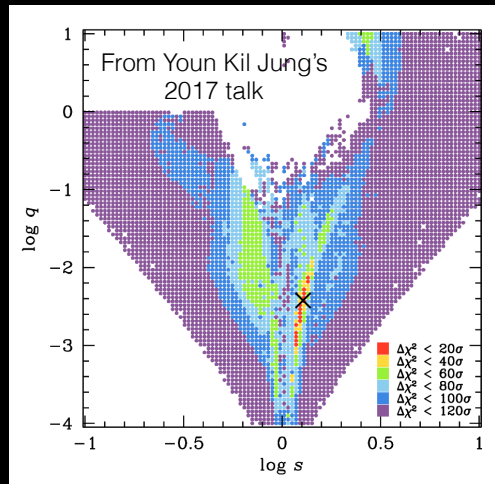
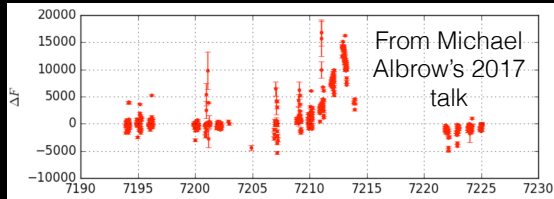
The goal of this talk:  
To start a conversation



Angie Wolfgang  
Penn State  
NSF Postdoctoral Fellow

# Problems and Opportunities:

“Closer” to light curves (i.e. more data-centric)

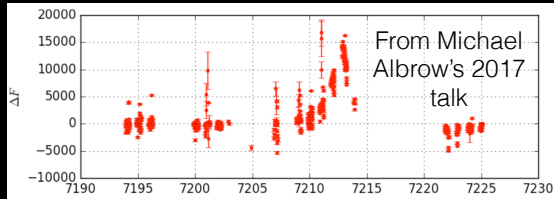


- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?
- Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?
- Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?
- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

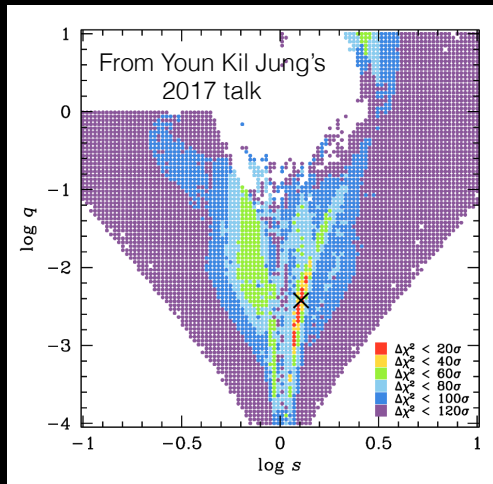
“Farther” from light curves (i.e. more model-centric)

# Problems and Opportunities:

“Closer” to light curves (i.e. more data-centric)

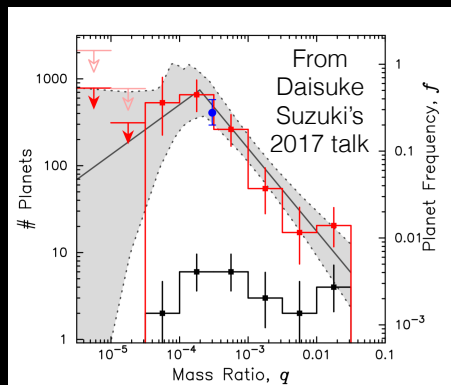


- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?



- Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?

- Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?

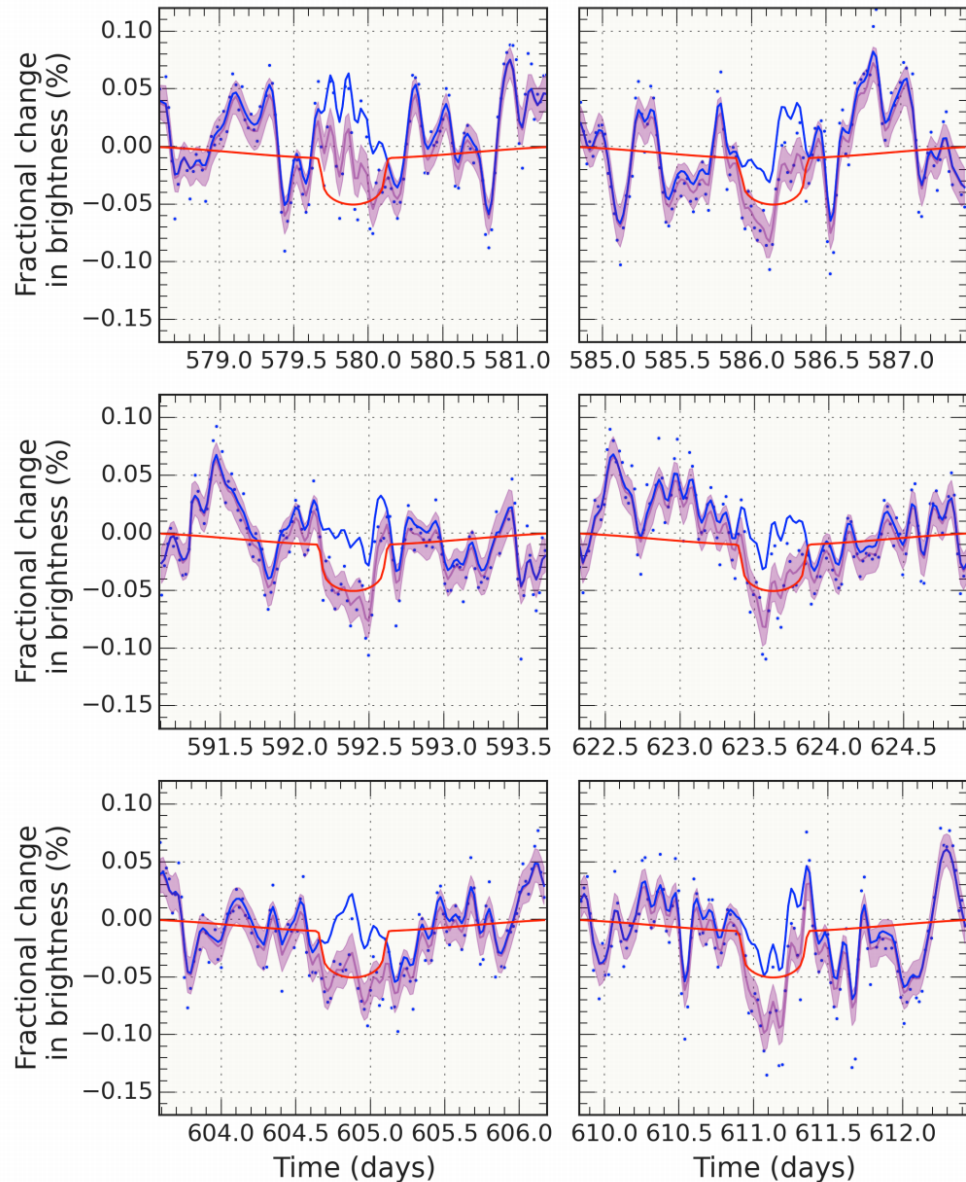


- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

“Farther” from light curves (i.e. more model-centric)

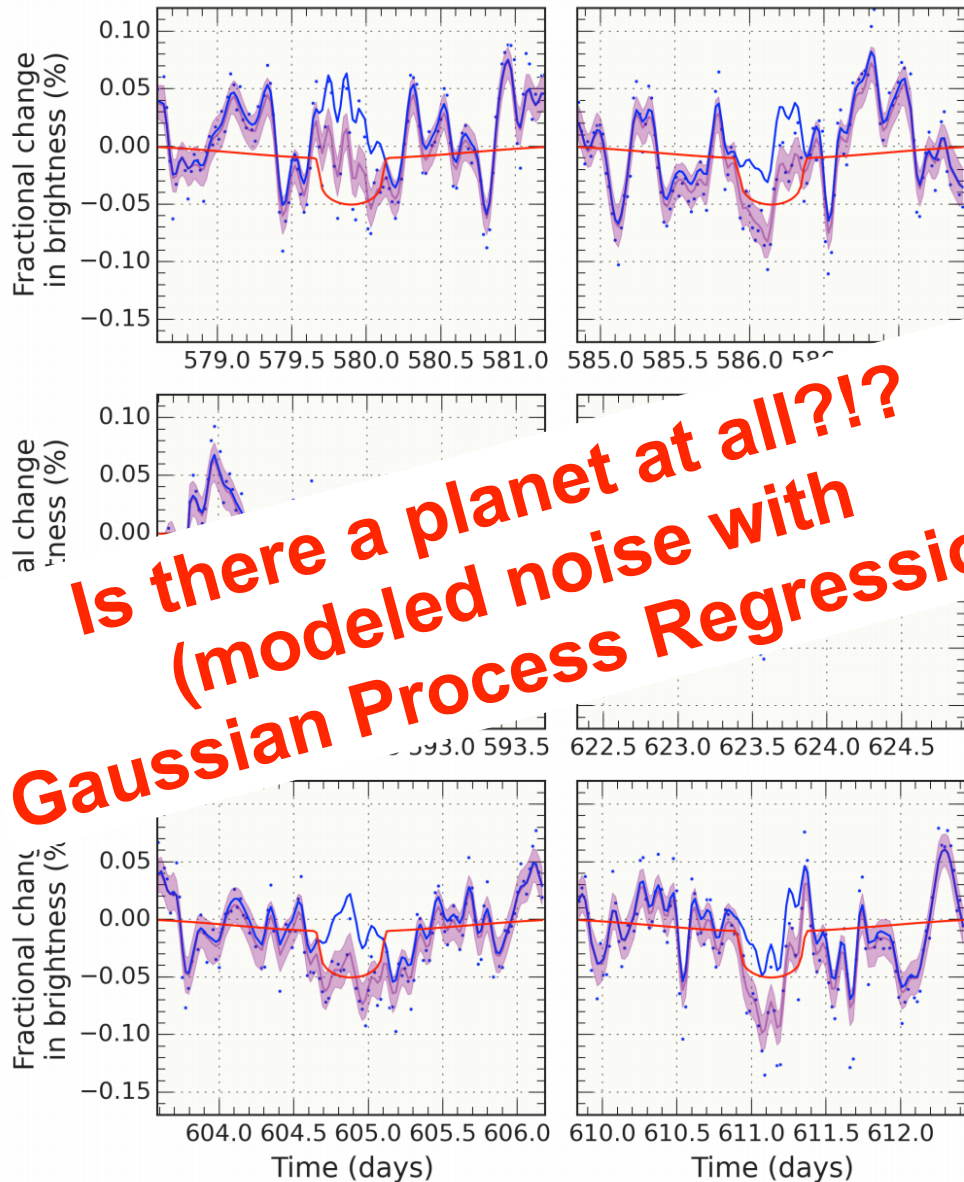
# Correlated Noise

Lesson learned from transits: Kepler-91 b (Barclay et al. 2015)



# Correlated Noise

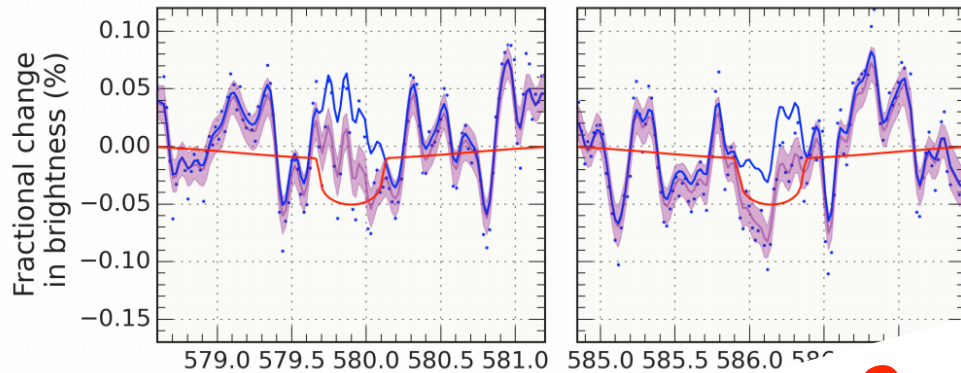
Lesson learned from transits: Kepler-91 b (Barclay et al. 2015)



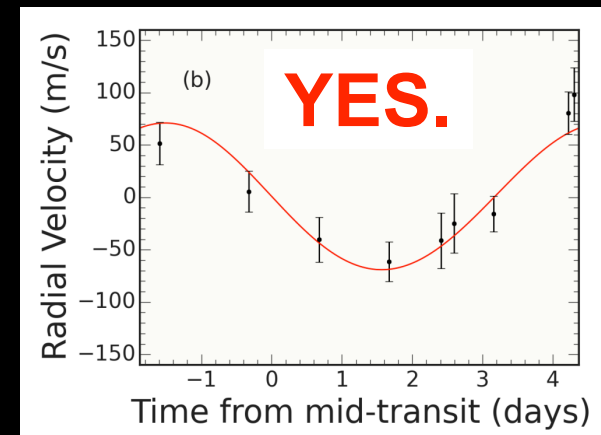
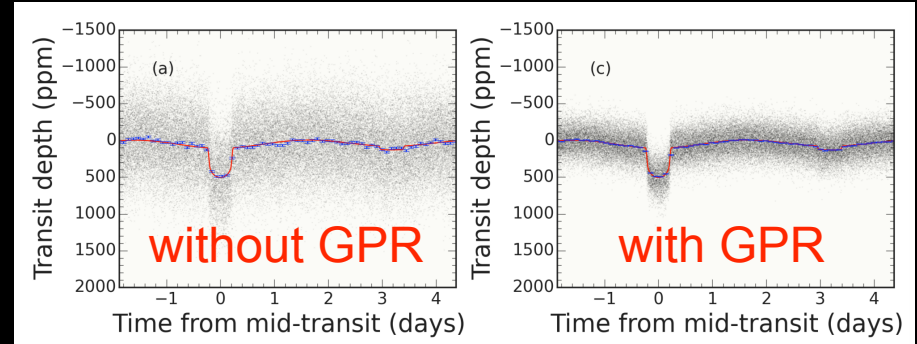
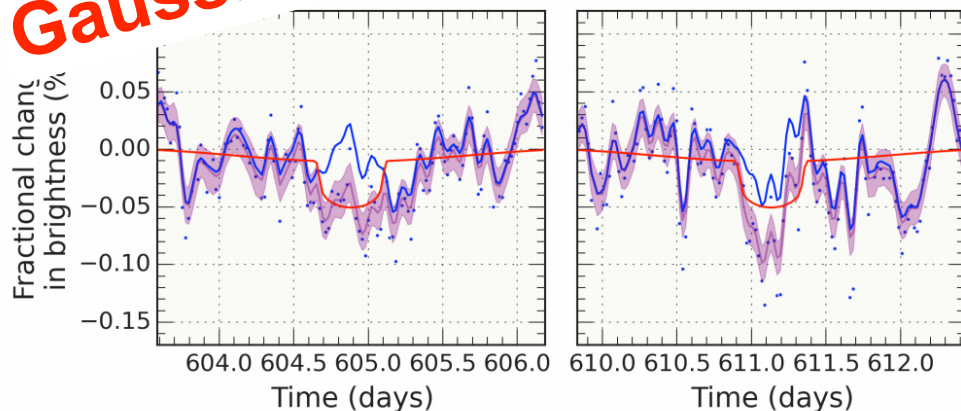


# Correlated Noise

Lesson learned from transits: Kepler-91 b (Barclay et al. 2015)

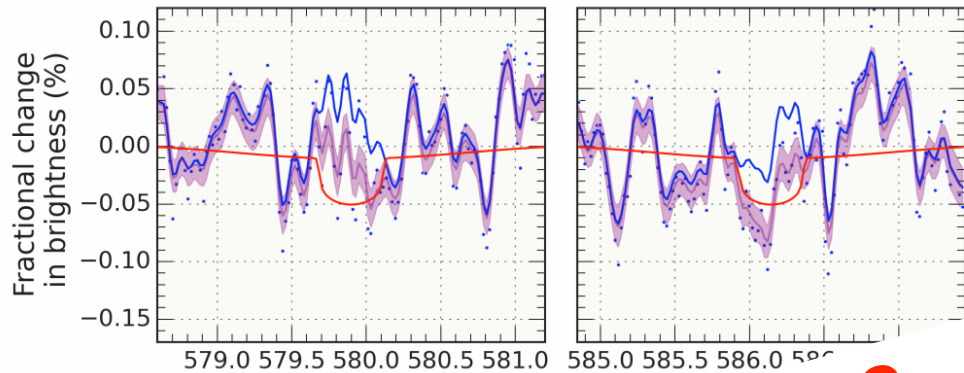


**Is there a planet at all?!?  
(modeled noise with  
Gaussian Process Regression)**

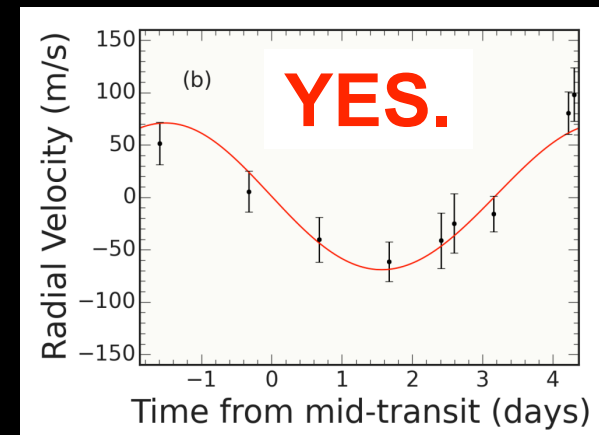
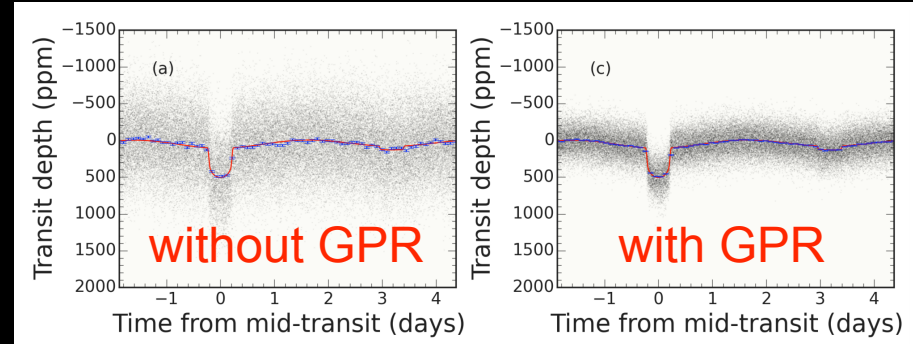
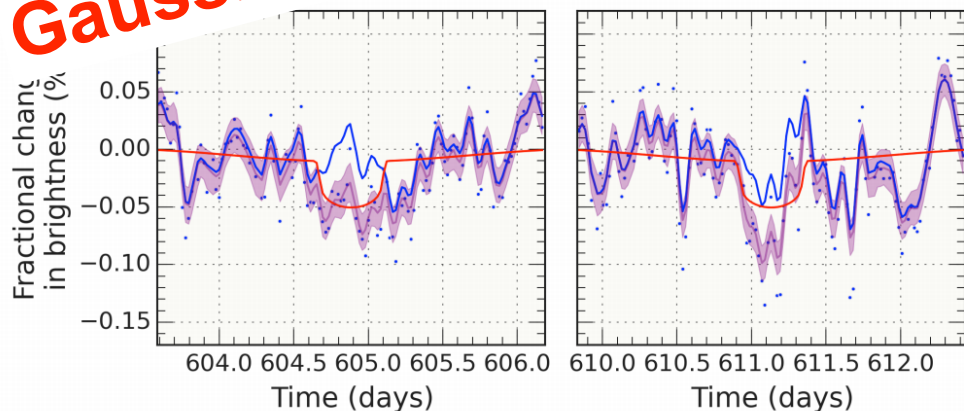


# Correlated Noise

Lesson learned from transits: Kepler-91 b (Barclay et al. 2015)



**Is there a planet at all?!?  
(modeled noise with  
Gaussian Process Regression)**



- 1) Correlated noise can be significant for red giants (common pulsing source)
- 2) Can drastically change your detections & parameter values!

# Correlated Noise

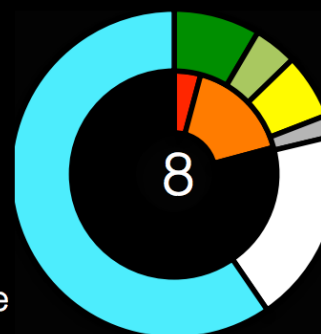
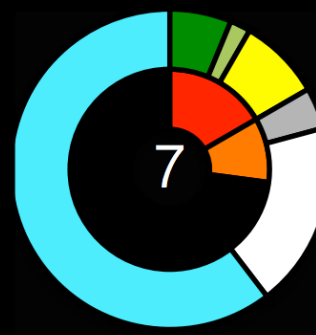
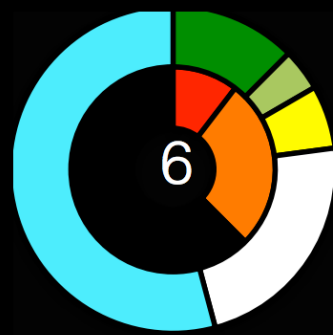
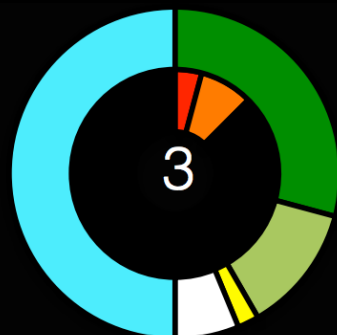
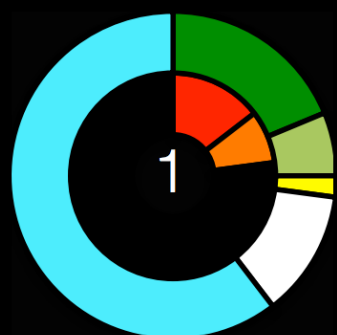
## Lesson learned from radial velocities: RV Fitting Challenge

|   | Team       | Techniques                                                                |
|---|------------|---------------------------------------------------------------------------|
| 1 | Torino     | Bayesian framework with Gaussian process to account for red noise         |
| 2 | Oxford     | Bayesian framework with Gaussian process to account for red noise         |
| 3 | M. Tuomi   | Bayesian framework with Moving Average to account for red noise           |
| 4 | P. Gregory | Bayesian framework with apodized Keplerians to account for red noise      |
| 5 | Geneva     | Bayesian framework with white noise                                       |
| 6 | A. Hatzes  | Pre-whitening                                                             |
| 7 | Brera      | Filtering in frequency space                                              |
| 8 | IMCCE      | Compressed sensing and filtering in frequency space (preliminary results) |

(Dumusque et al. 2017)

1) A Bayesian framework + red noise model produces more reliable and complete detections.

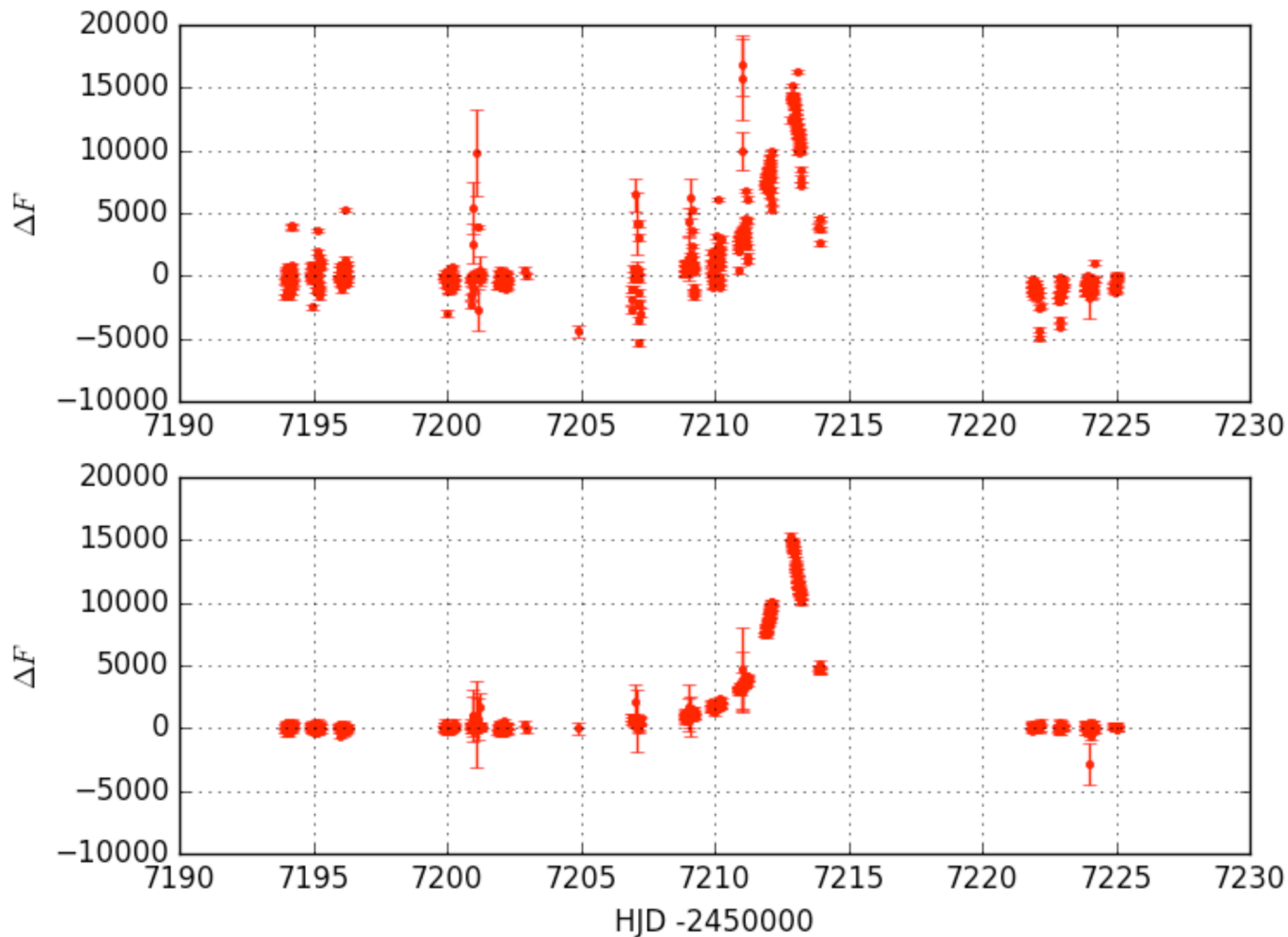
2) Inflexible noise models more often lead to inaccurate parameter values.





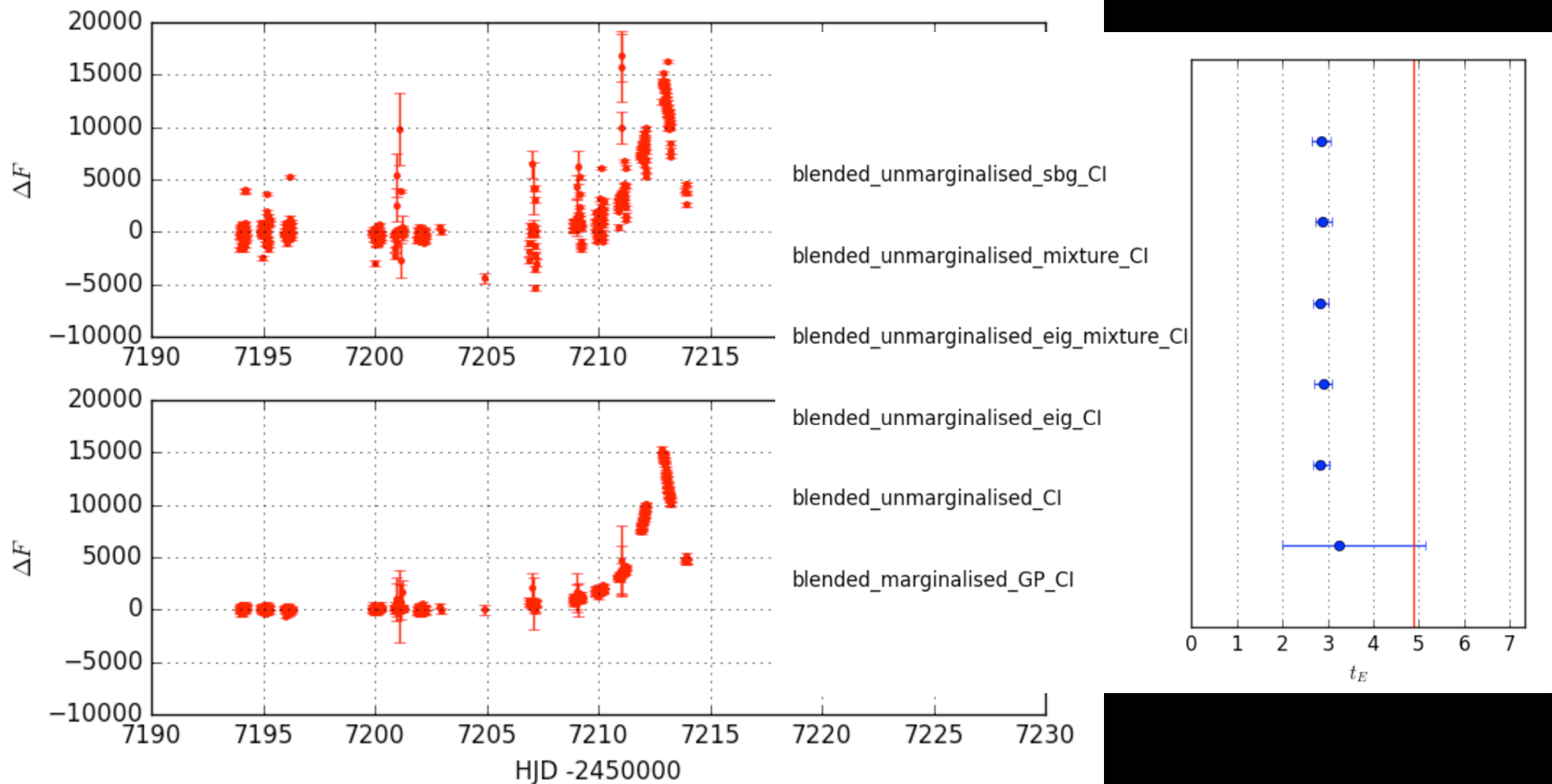
# Correlated Noise

Effect on microlensing parameters (from Albrow's 2017 talk)



# Correlated Noise

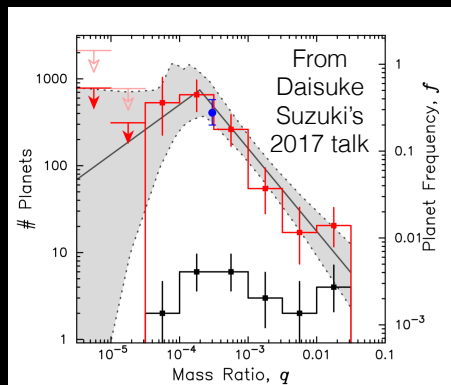
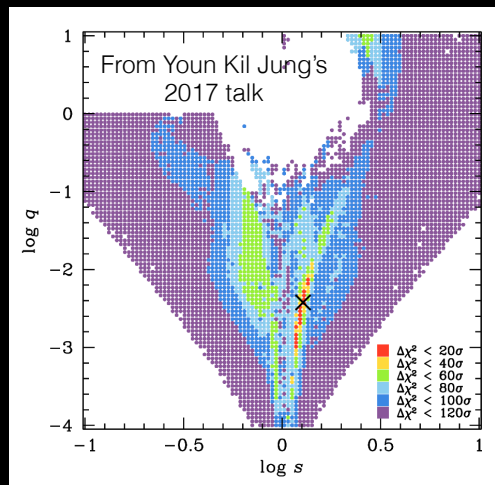
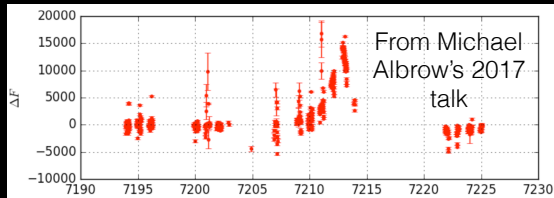
Effect on microlensing parameters (from Albrow's 2017 talk)



Red noise can seriously mislead you, too!! Test more than  $t_E$ !

# Problems and Opportunities:

“Closer” to light curves (i.e. more data-centric)

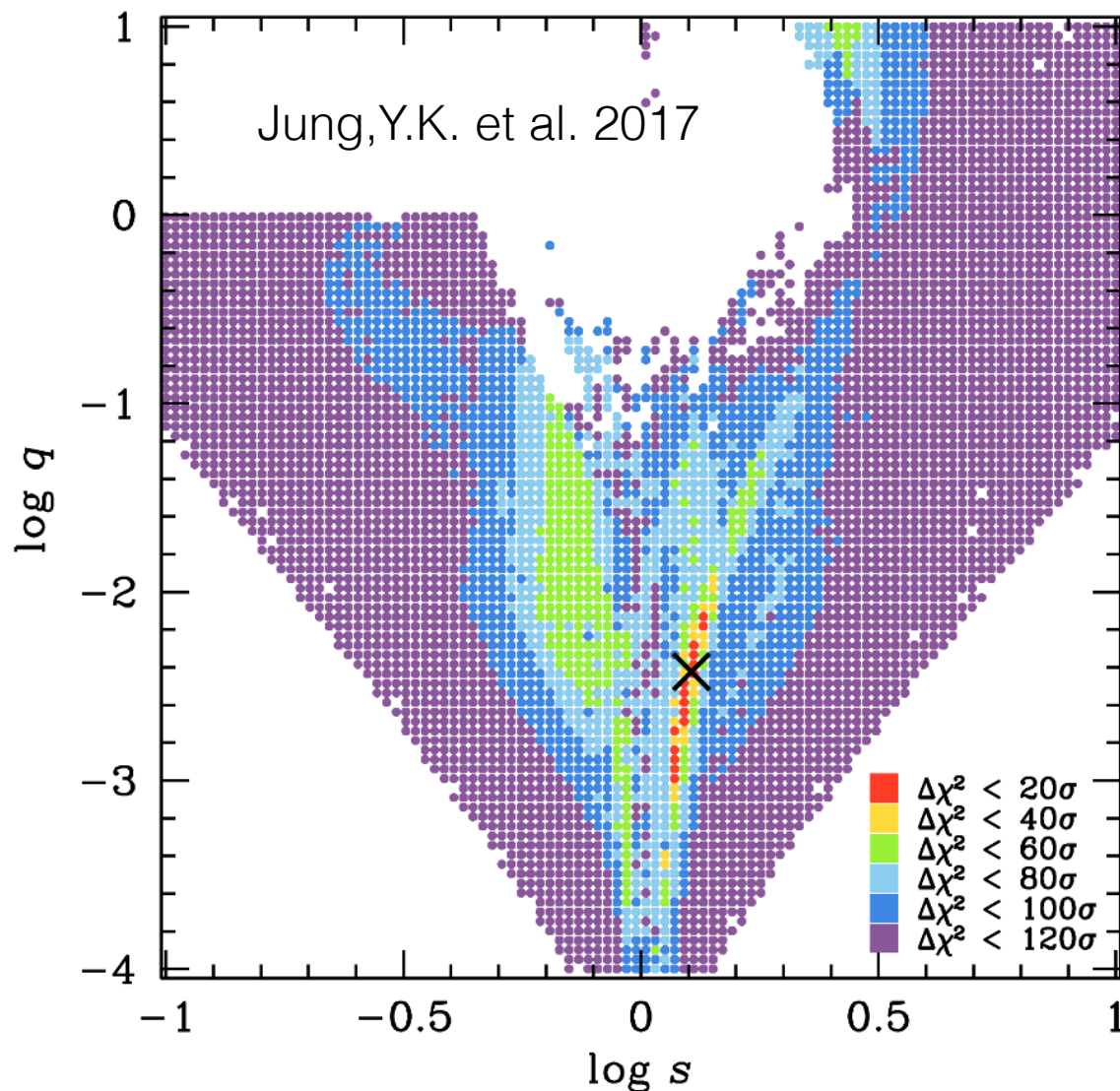


- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?
- Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?
- Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?
- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

“Farther” from light curves (i.e. more model-centric)

# Statistical Emulation

of computationally expensive astrophysical simulations



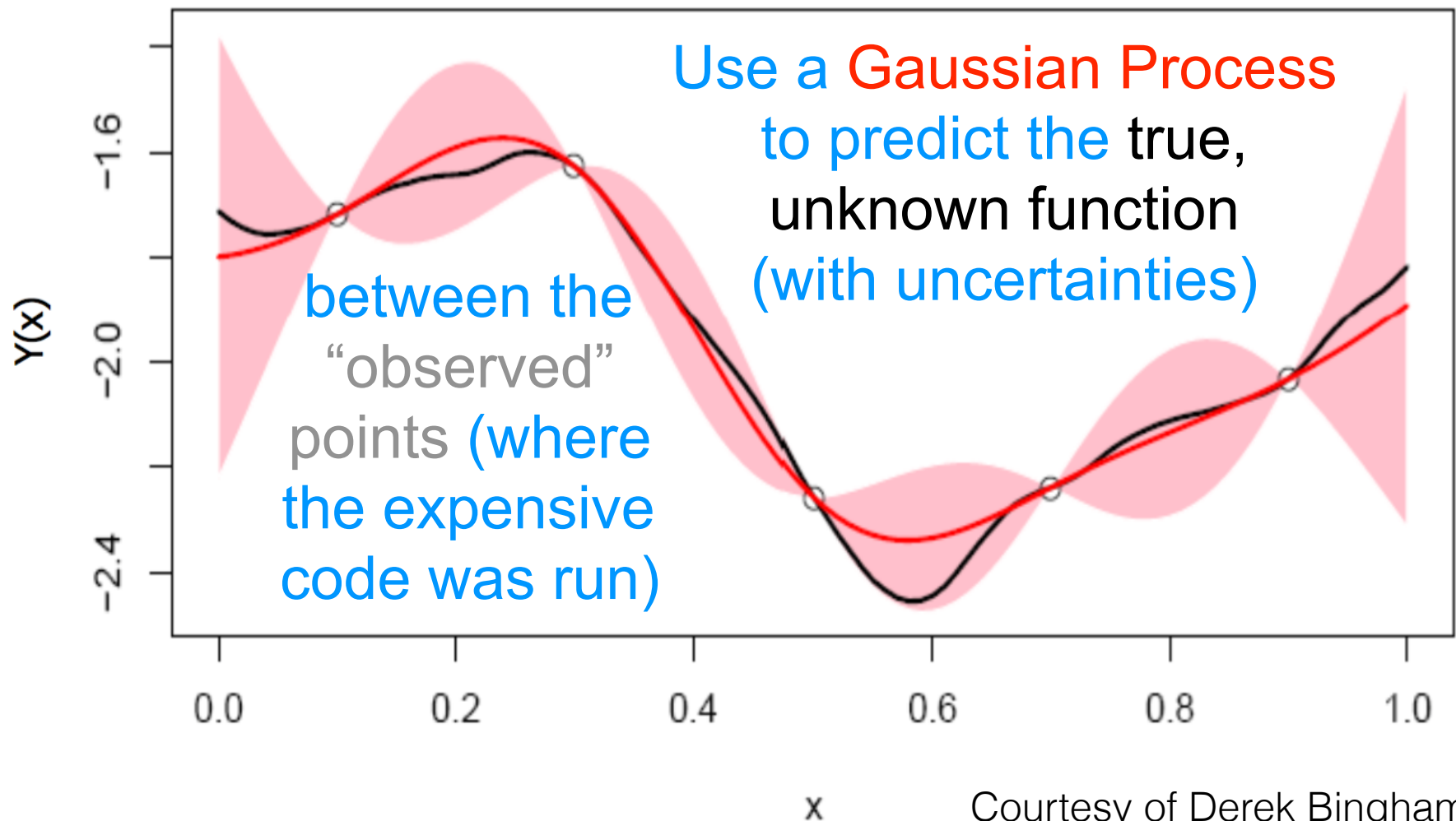
Is there a more efficient way to compute the grids of  $q$ ,  $s$ , and  $\alpha$ ?

**YES: statistical emulation**  
... plus it allows you to interpolate the grid for free  
... plus that interpolation is probabilistic

# Statistical Emulation

## A 1-dimensional conceptual example

True function, emulated mean function and 95% prediction intervals



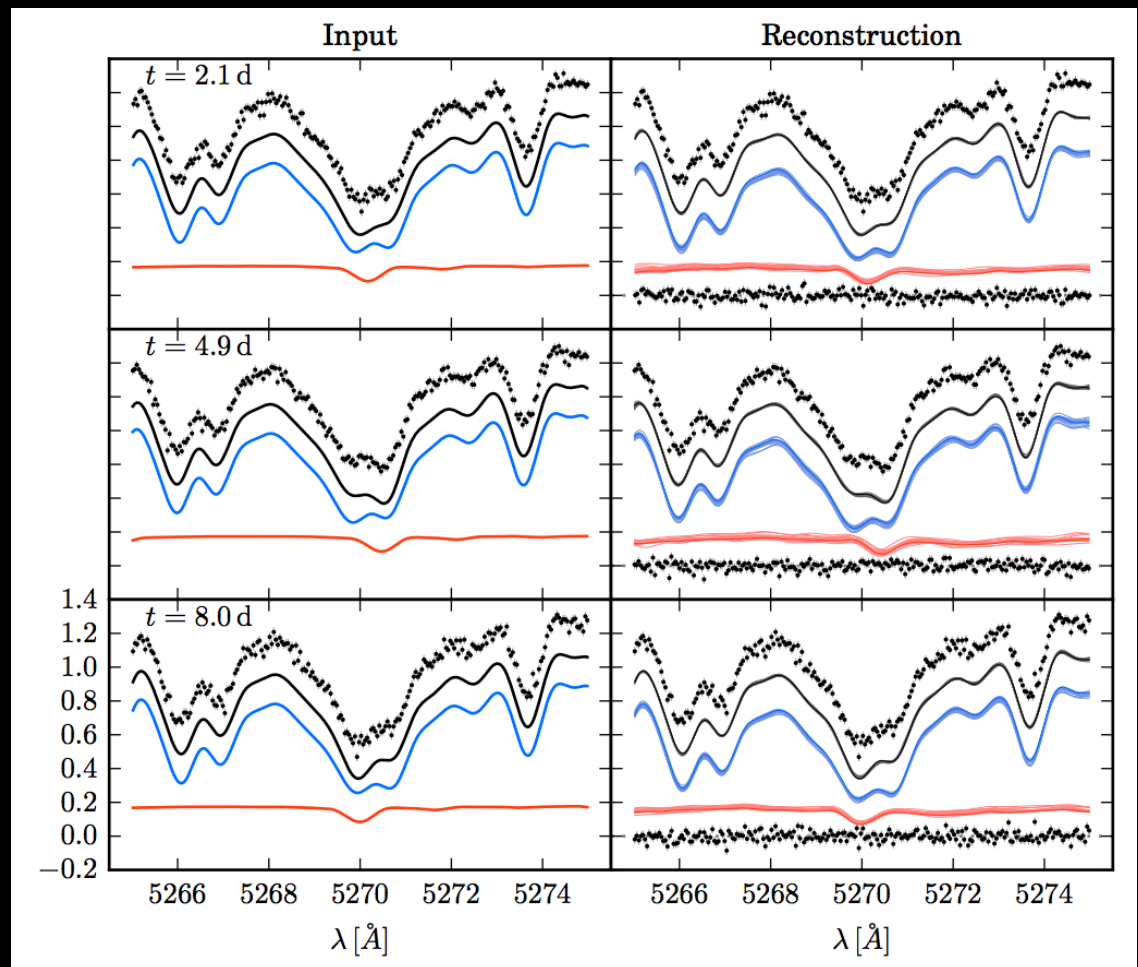
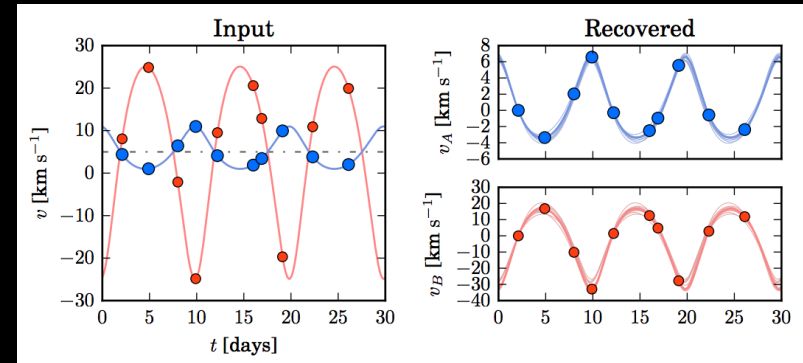


# Statistical Emulation

A multi-dimensional  
astrophysical example  
(Czekala et al. 2017)

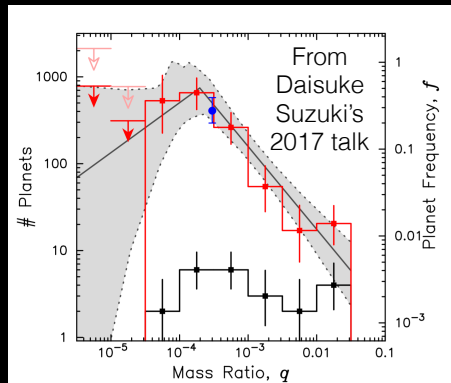
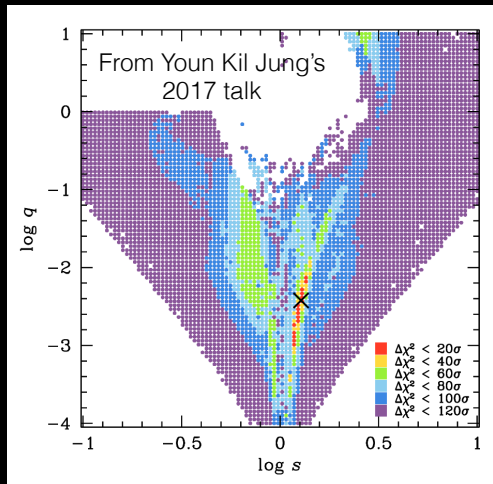
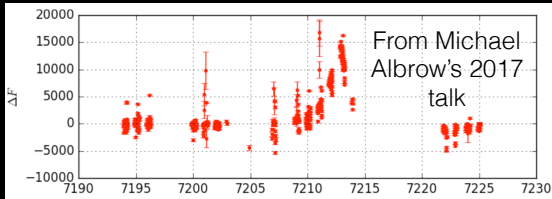
Simulated the time-  
varying spectrum of an  
SB1 binary star system

Used a Gaussian  
Process emulator to  
model the spectrum,  
predict/interpolate the  
radial velocities of each  
component, and infer  
both stars' masses.



# Problems and Opportunities:

“Closer” to light curves (i.e. more data-centric)

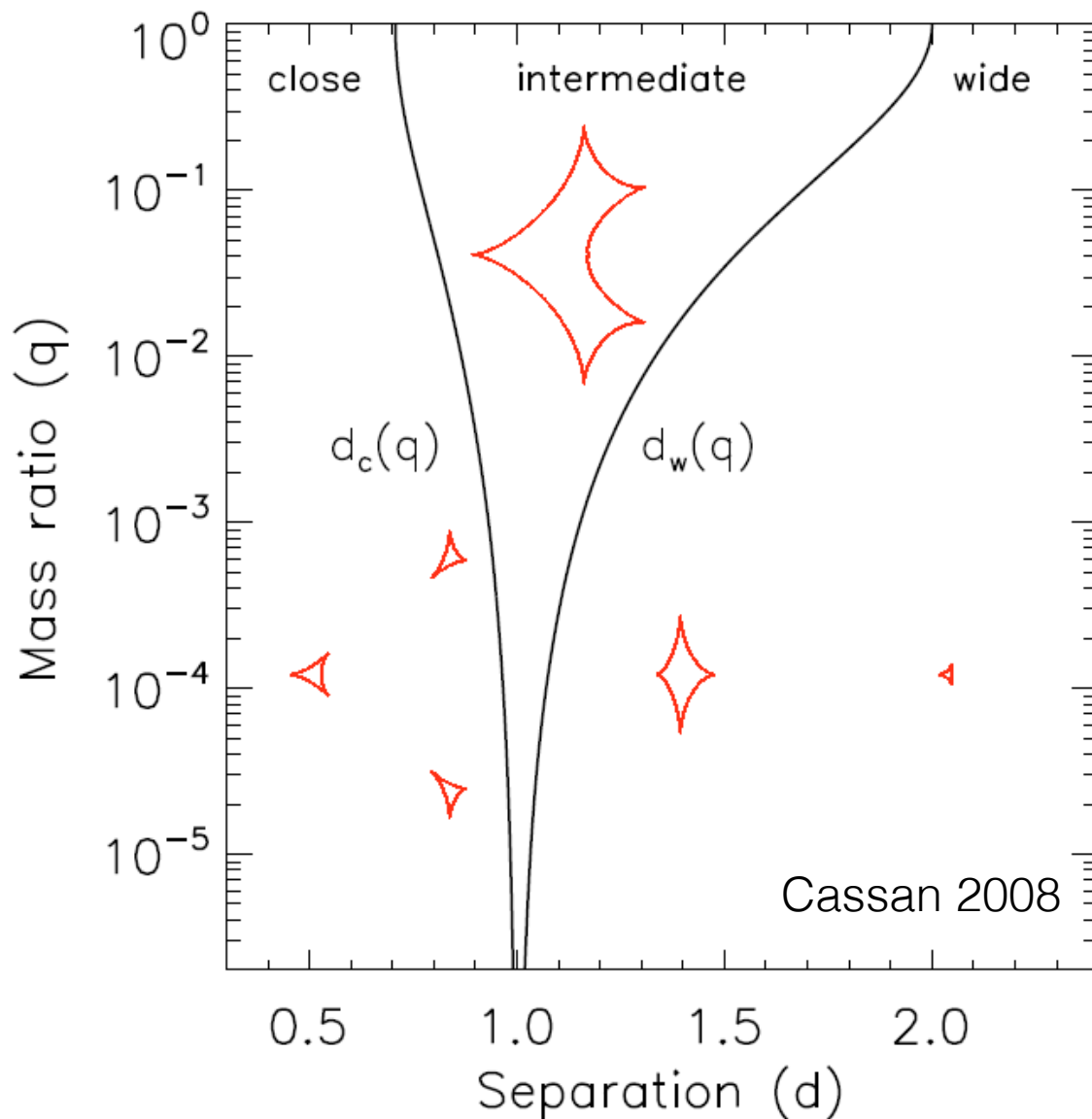


- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?
- Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?
- Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?
- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

“Farther” from light curves (i.e. more model-centric)

# Model Comparison

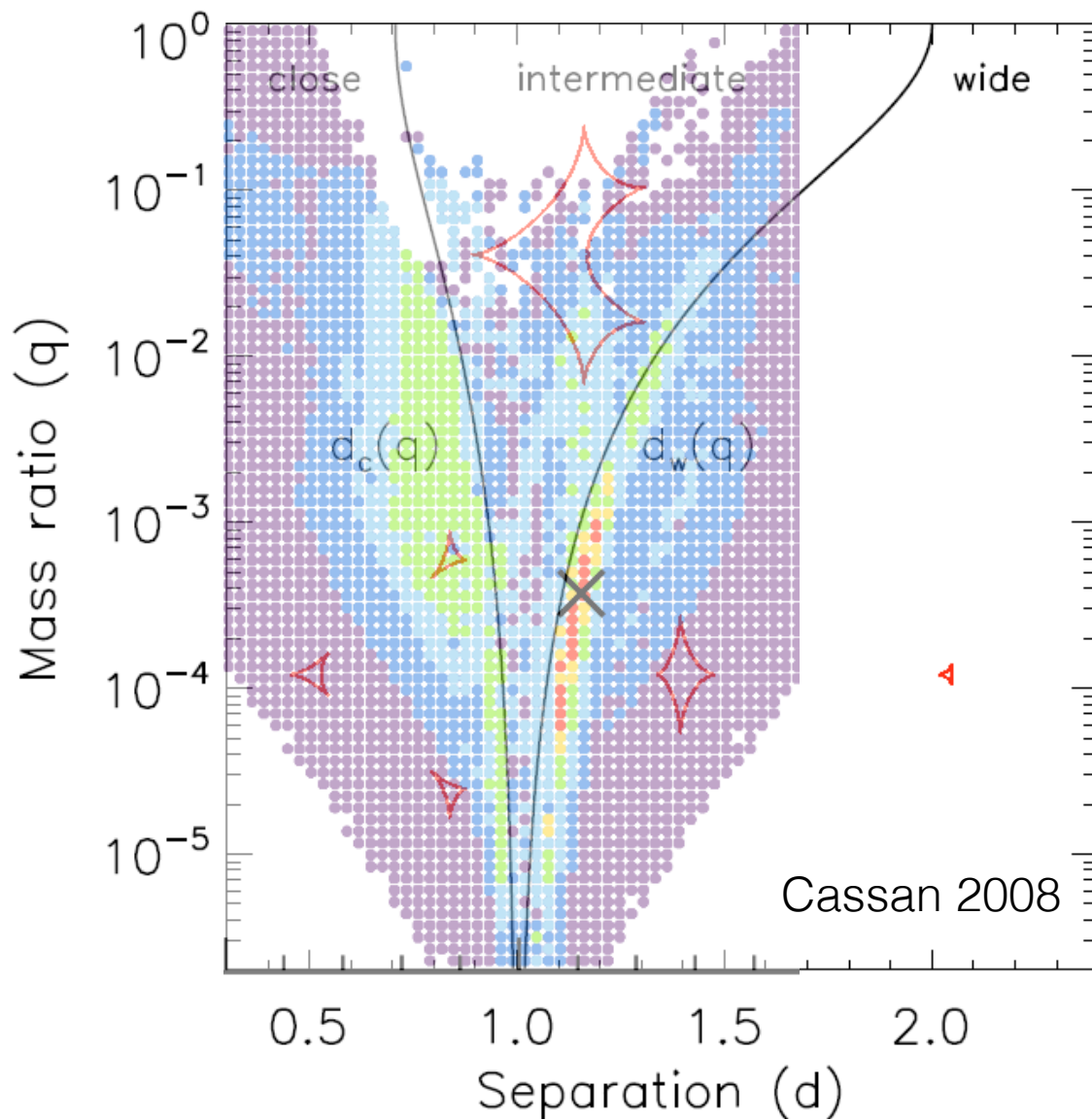
Which caustic topology best fits the data?



Actually have 3 separate models to fit to the data ...

# Model Comparison

Which caustic topology best fits the data?

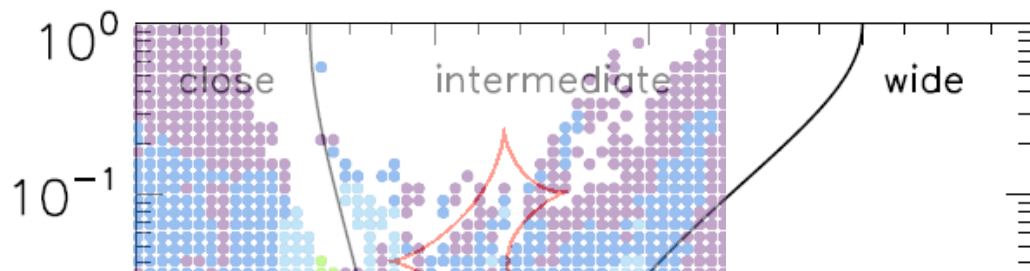


Actually have 3  
separate models  
to fit to the data ...  
which leads to  
multimodal  
likelihood spaces

Is it better to identify  
a single point as a  
best fit, or to  
integrate over the  
parameters for that  
model to identify  
most likely topology?

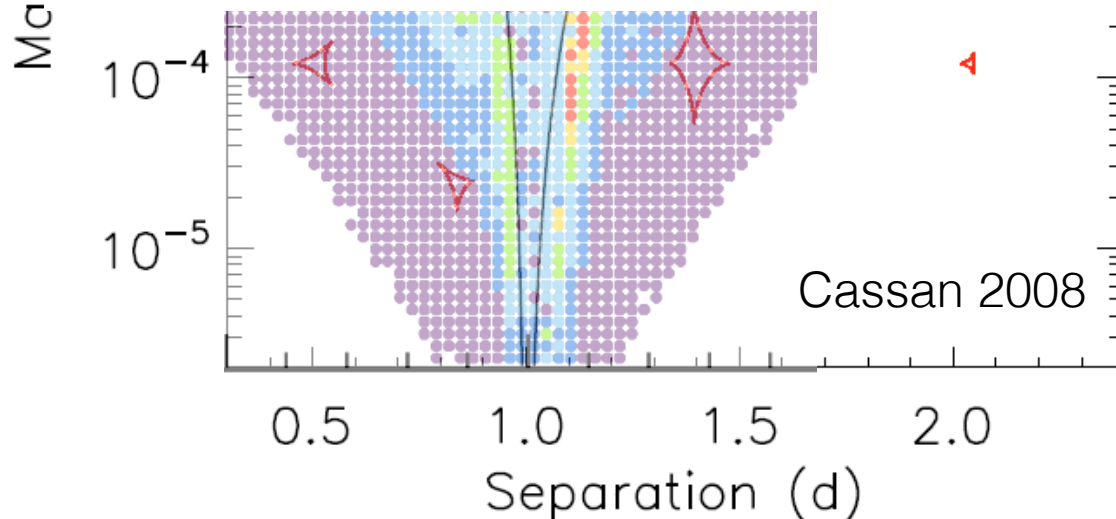
# Model Comparison

Which caustic topology best fits the data?



Actually have 3 separate models to fit to the data ...

**How can you do model comparison?  
See Ben Nelson's talk on Saturday  
with lessons learned from RV.**

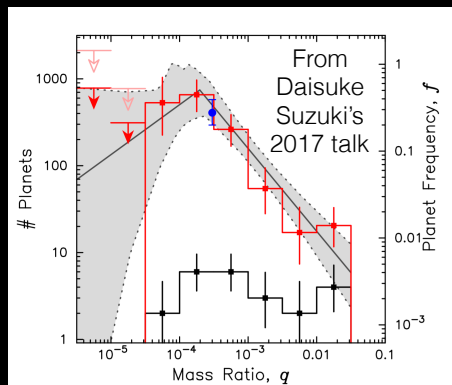
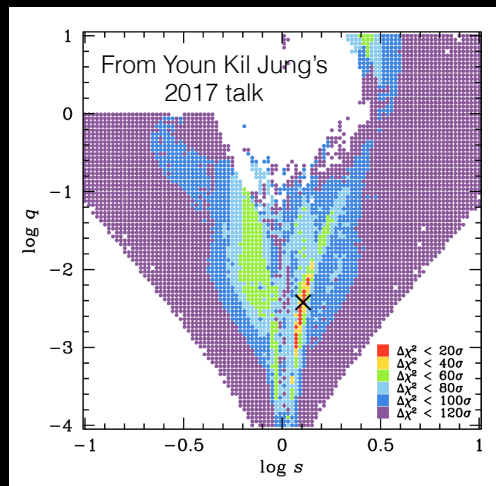
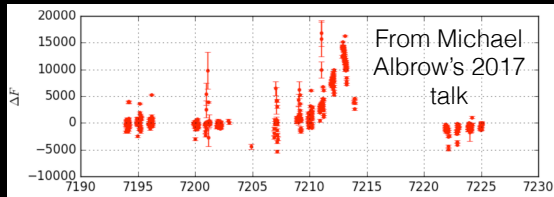


a single point as a best fit, or to integrate over the parameters for that model to identify most likely topology?



# Problems and Opportunities:

“Closer” to light curves (i.e. more data-centric)



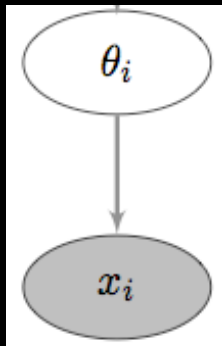
- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?
  - Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?
  - Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?
- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

“Farther” from light curves (i.e. more model-centric)

# Microlensing Populations

## Within a Hierarchical Bayesian Framework

What do I mean?



Individual  
Parameters

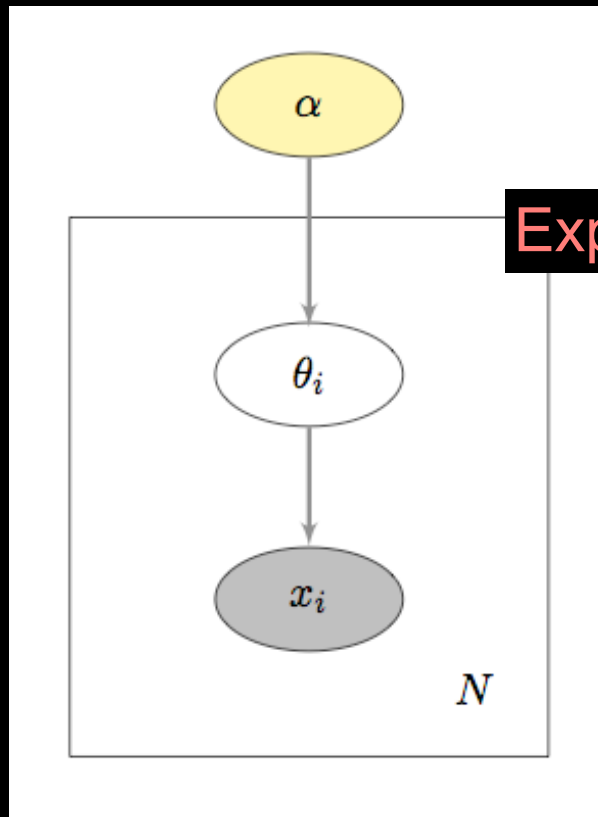
(likelihood  $\rightarrow$  MCMC)

Observables

# Microlensing Populations

## Within a Hierarchical Bayesian Framework

What do I mean?



Population  
Parameters

Expand your likelihood!

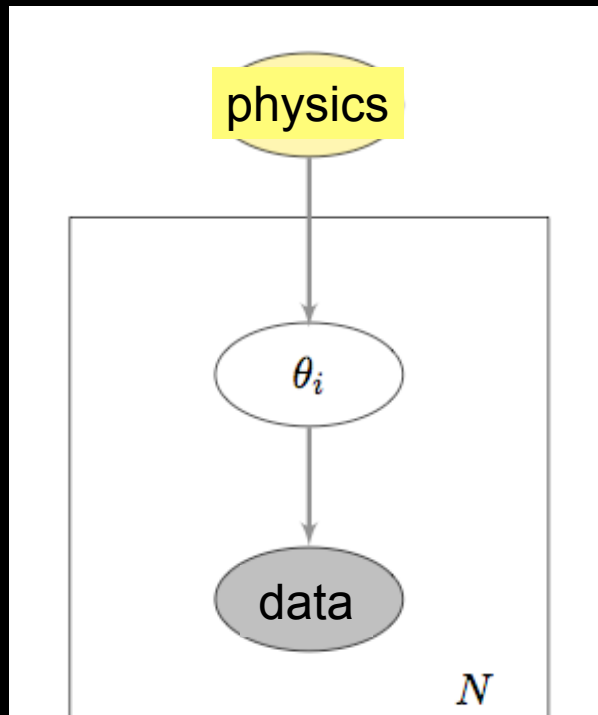
Individual  
Parameters

Observables

# Microlensing Populations

## Within a Hierarchical Bayesian Framework

What do I mean?



Population  
Parameters

Individual  
Parameters

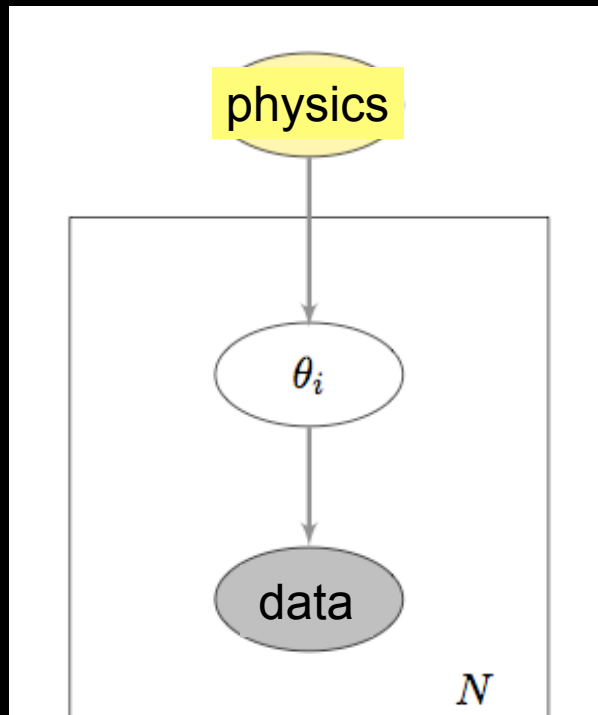
Observables

Structure helps constrain posteriors:  
Wolfgang & Lopez, 2015

# Microlensing Populations

## Within a Hierarchical Bayesian Framework

What do I mean?



Population  
Parameters

Individual  
Parameters

Observables

Why do we need it?

- Parameter **inference with uncertainties**
- Naturally deals with large measurement uncertainties and **upper limits**
- Can account for **selection effects**  
\*within\* the inference
- **Simultaneous posteriors** on individual and population parameters
- **Directly ties theory** to observations
- Framework for **model comparison**

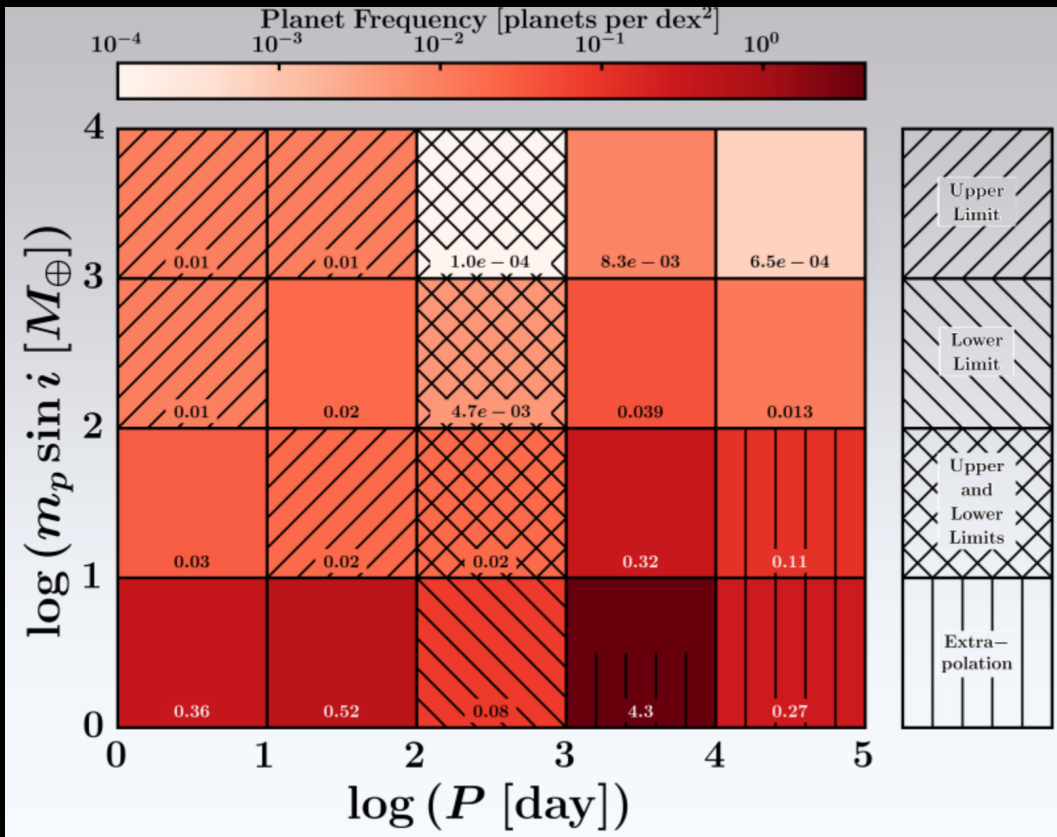
Structure helps constrain posteriors:  
Wolfgang & Lopez, 2015

Still use MCMC: “Just” adding another layer of probabilistic structure



# Microlensing Populations

What would a hierarchical Bayesian framework add?

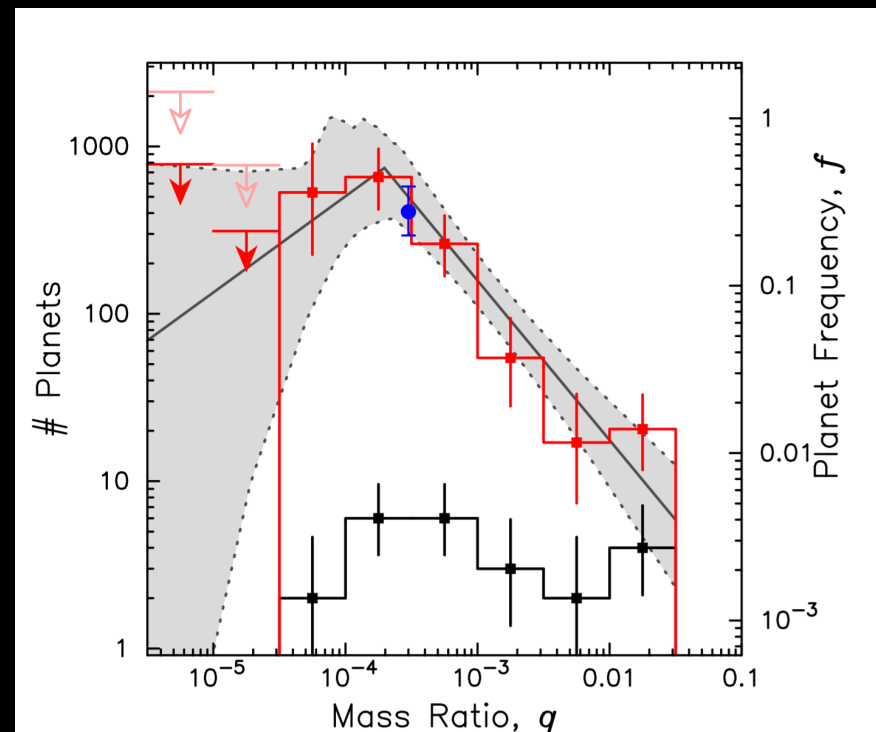


Clanton & Gaudi 2017

2) Can incorporate all degenerate solutions probabilistically.

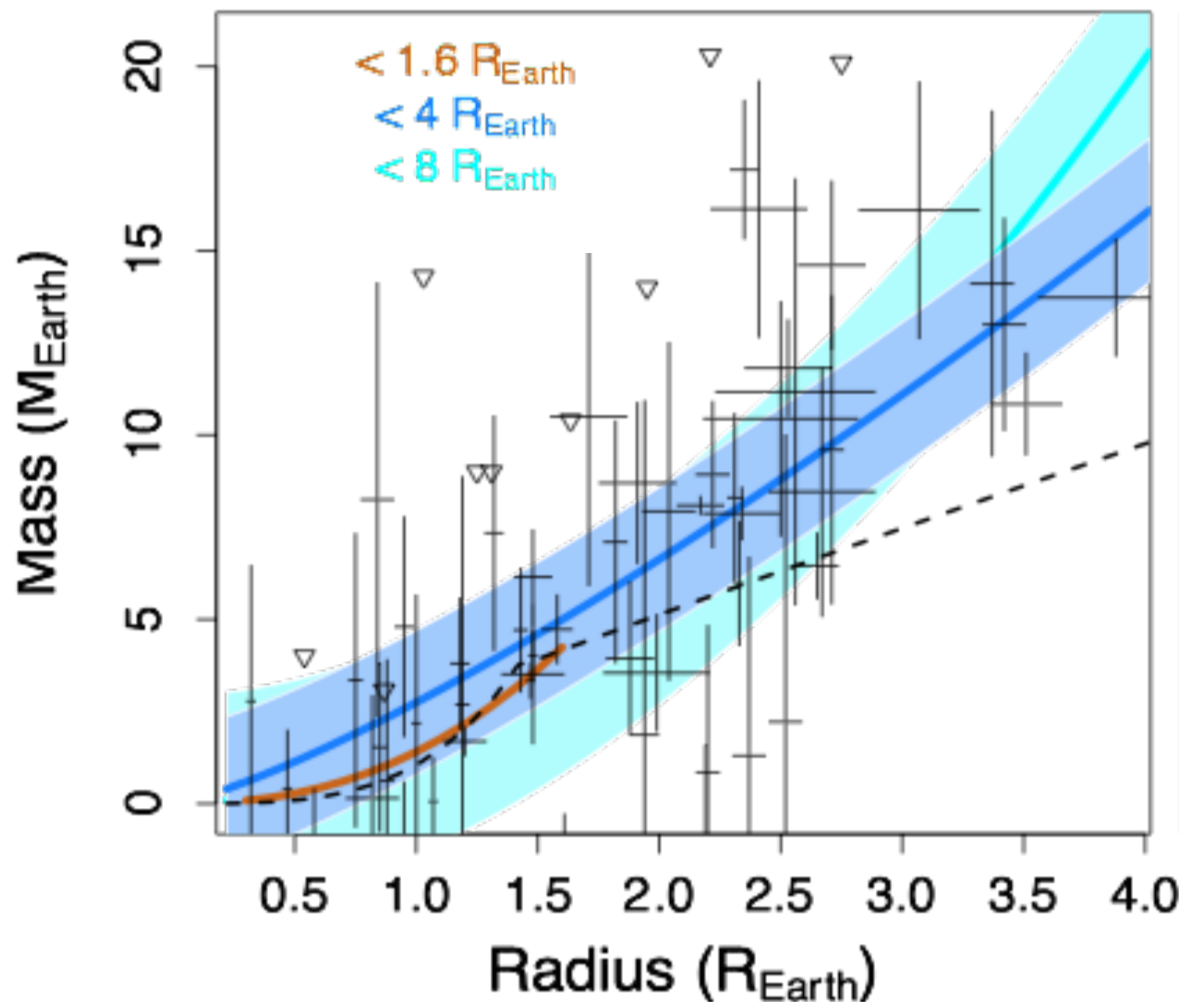
Suzuki et al. 2017

1) Uncertainties in population can easily and self-consistently incorporate uncertainties (including correlated) on microlensing, physical, \*and\* nuisance parameters



# Probabilistic Populations

Examples from Kepler (Wolfgang et al. 2015, 2016):  
sub-Neptune compositions and mass-radius relations



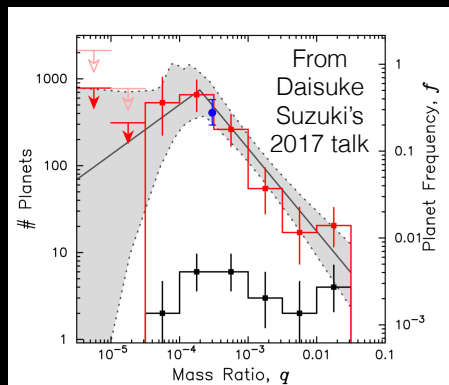
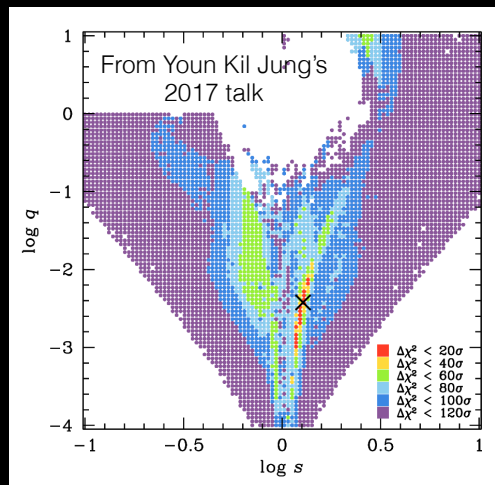
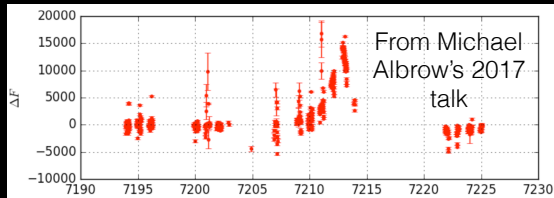
Allows for a distribution  
of masses at a given radius  
as is motivated by  
observations and theory

Can distinguish between  
scatter due to  
measurement uncertainty  
and astrophysical scatter  
in the planet population

No binning necessary;  
also includes upper limits

# Thank you - Thoughts?

“Closer” to light curves (i.e. more data-centric)



- Correlated noise in light curves - how does a more advanced noise model change your inferred parameter values?
- Computationally expensive ray-shooting codes - can statistical emulation speed up microlensing parameter estimation?
- Likelihood of data given parameters can be multimodal, and caustic topographies are not continuous - how to choose the best model?
- Population analyses depend on uncertain and correlated parameter values for individual events - how to account for these in a self-consistent probabilistic framework?

“Farther” from light curves (i.e. more model-centric)