1. \[ 2018 \times 9996 = 20171928 \]

2. **Claim:** 19. In fact, the "MakeMyDay" procedure does not change the maximum difference between two numbers on the list. Suppose our list is \{a, b, c\} with \(a < b < c\). The maximum difference between the largest and the smallest number is \(c - a\). The "MakeMyDay" operation creates \{b + c, a + c, a + b\}.

Since \(a < b\), we know that \(a + c < b + c\). Since \(b < c\), we also know that \(a + b < a + c\). Combining these two inequalities, we have \(a + b < a + c < b + c\).

The maximum difference between any number is \((b + c) - (a + b)\) or \(c - a\). So the same as the one we started with. For the initial list of \{20, 1, 8\}, the maximum difference will always be 19.

3. Inscribe in a rectangle, where \(\angle CBP = 30\text{deg}\).

Since \(BC = 2\), we get \(CP = 1\), and \(BP = \sqrt{3}\). So \(AP = 1 + \sqrt{3}\).

Then \(\angle DCQ = 60\text{deg}\), so \(CQ = 1/2\), \(PQ = 3/2\), \(DQ = (\sqrt{3})/2\).

Finally, \(DR = 1 + \sqrt{3} - (\sqrt{3})/2 = \frac{2 + 2\sqrt{3} - \sqrt{3}}{2} = 1 + (\sqrt{3})/2\), and \(ER = 1/2\).

The area of the pentagon is then \[ \frac{AP \times PQ}{2} - \frac{1}{2}(BP \times CP + CQ \times DQ + DR \times ER) \]
\[ = \frac{1}{4}(5 + 3\sqrt{3}) \approx 2.549 \text{ units}^2 \]

4. Let \(b_i\) and \(g_i\) be the numbers of boys and girls on board the tram, respectively, at stop \(i\). Note that \(b_0\) and \(g_0\) are the numbers of boys and girls on board the tram, respectively, at the start of the trip.

At stop 1, \(b_1 = b_0 + g_0/3\), \(g_1 = 2g_0/3\).

Similarly, at stop 2, \(b_2 = 2b_1/3\), \(g_2 = g_1 + b_1/3\).

Using above formulas, for \(b_1\), \(g_1\) leads to \(b_2 = 2b_0/3 + 2g_0/9\), \(g_2 = 7g_0/9 + b_0/3\). From \(b_2 = g_0\) this yields \(b_0 = 7g_0/6\).

As \(b_2 + 2 = g_2\), we have \(g_0 = 1/5(3b_0 + 18)\).

We can now solve for \(g_0\) to get \(g_0 = 12\) and then \(b_0 = 14\).

5. We have \(b \leq a + 1\), \(c \leq b + 1\), \(a \leq c + 1\), so that \(c - 1 \leq b \leq a + 1 \leq c + 2\), so \(a\) in \{\(c - 2\), \(c - 1\), \(c + 1\)\}. Now a case bash yields 10 solutions:

\((1,2,3),(3,1,2),(2,3,1),(3,4,5),(5,3,4),(4,5,3),(2,1,1),(1,2,1),(1,1,2),(1,1,1)\).
Suppose \( a \) and \( b \) are both positive. Then \( a(x - a)^2 \geq 0 \) and \( b(x - b)^2 \geq 0 \) with the first being zero for \( x = a \) and the second being zero for \( x = b \).

Thus \( a(x - a)^2 + b(x - b)^2 = 0 \) only when \( x = a = b \).

If \( a \) and \( b \) are both negative, then the reasoning is similar.

If \( a \) and \( b \) have opposite signs, we rewrite the equation as follows:

\[
(a(x - a))^2 + (b(x - b))^2 = (a + b)x^2 - 2(a^2 + b^2)x + (a^3 + b^3) = 0
\]

Then its discriminant \( \Delta \) is

\[
\Delta = 4(a^2 + b^2)^2 - 4(a + b)(a^3 + b^3)
\]

\[
= -4ab(-2ab + b^2 + a^2)
\]

\[
= -4ab(a - b)^2 < 0,
\]

is negative unless \( b = -a \).

We have

\[
N = (10 - 1) + (10^2 - 1) + \ldots + (10^{2018} - 1) = 111 \ldots 10^{2018} - 2018
\]

\[
= 111 \ldots 100000 + (11110 - 2018)
\]

\[
= 111 \ldots 100000 + 9092.
\]

Hence in decimal representation there are 2014 ones.

The area of the \( M \)-region is area of rectangle \( ABCD \) - area \( \Delta ACE \) as triangle \( \Delta ACE \) is the overlap.

| \( ABCD \) | = 12 x 18 = 216.
| \( AC \) | is the diagonal of the rectangle,
so \( AC = \sqrt{12^2 + 18^2} = 6\sqrt{13} \).

The height of \( \Delta ACE \) is \( \overline{EF} \), and equals \( \overline{FC} \times \tan \angle DAC \)

\[
|\overline{FC}| = \frac{3\sqrt{13}}{2}
\]
and \( \tan \angle DAC = \frac{12}{18} = \frac{2}{3} \).

So \( |\Delta ACE| = \frac{1}{2} |\overline{AC}| \times |\overline{EF}| = 78. \)

Area = 216 - 78 = 138.

OR . . .

With \( \Delta ADC = \text{half rectangle } ABCD = 108 \) then remaining shaded area = \( \Delta ABE \), and since side \( AB = 12 \), we can find \( BE \) from \( 12 \times \tan(\angle BAE) \), where \( \angle BAE \) is from \( \angle BAC = \angle DAC \) (i.e. from \( \tan^{-1}(\frac{12}{18}) \)).

So \( BE = 5 \), and smaller \( \Delta ABE = 30 \), so combined shaded area of \( \Delta ADC + \Delta ABE = 108 + 30 = 138. \)

OR . . .

We have \( \overline{AE} (= \overline{EC}) + \overline{ED} = 18 \), so set up the equation with \( ED = \overline{EB} = x \) so that

\[
x^2 + 12^2 = (18 - x)^2
\]

\[
144 = 324 - 36x
\]
Solves \( x = 5 = \overline{ED} \).

Therefore the shaded area

\[
= \frac{1}{2} (12 \times 18) + \frac{1}{2} (5 \times 12) = 138.
\]
If after her second move Alice does not win, then Bob wins with his second move. Indeed, in this case there are two numbers $a$ and $b$ on the board with the same parity. Bob wins with writing $\frac{1}{2}(a + b)$.

We just need to show how Bob chooses his first move. If Alice chooses $a \leq 1009$, then Bob chooses the number in the set $\{2017, 2018\}$ whose parity is different from $a$. And, if Alice chooses $a \geq 1010$, then Bob chooses $b = 1$ or $2$, the one whose parity is different from $a$.

Then Alice cannot win with her second move since $\frac{1}{2}(a + b)$ is not an integer.

Firstly, let us show that $n = 15$ is possible. Indeed, we can have the sequence 

$$+1 + 1 + 1 + 1 + 1 - 1 - 1 - 1 - 1 - 1 + 1 + 1 + 1 + 1 + 1:$$

Let us prove that $n$ cannot be larger. Suppose $n \geq 16$ and our sequence is $x = x_1 x_2 x_3 \ldots x_n$.

Without loss of generality suppose $x_1 = 1$.

Then, as the sum of every 10 neighbouring numbers is 0, we have $x_{11} = 1$.

Thus we have $x = 1x_2 x_3 \ldots x_{10} 1x_{12} x_{13} \ldots x_n$.

We claim that $x_{12} = \ldots = x_n = 1$.

Indeed, among these, -1 cannot follow 1 since then we will have the sum of 12 consecutive terms of the sequence, which ends with these +1 and -1 being 0. If $n \geq 16$, then we have at least 6 ones at the end of the sequence which it then makes impossible to have the sum of the last 10 terms to be 0.