2018 AUCKLAND MATHEMATICAL OLYMPIAD

Questions

• Write all your working and solutions below the question.
• You are expected to show how you obtained your solution for each question, as correct answers (without working) will be awarded 1 mark only.
• Markers will mark only one question at a time per candidate.

Junior Division

1. Find a multiple of 2018 whose decimal expansion’s first four digits are 2017.

2. Starting with a list of three numbers, the “Make-My-Day” procedure creates a new list by replacing each number by the sum of the other two.
   For example, from {1, 3, 8} “Make-My-Day” gives {11, 9, 4} and a new “Make-My-Day” leads to {13, 15, 20}.
   If we begin with {20, 1, 8}, what is the maximum difference between two numbers on the list after 2018 consecutive “Make-My-Day”s?

3. Consider the pentagon below. Find its area.

4. A vintage tram departs a stop with a certain number of boys and girls on board.
   At the first stop, a third of the girls get out and their places are taken by boys.
   At the next stop, a third of the boys get out and their places are taken by girls.
   There are now two more girls than boys and as many boys as there originally were girls.
   How many boys and girls were there on board at the start?

5. Find all possible triples of positive integers, \(a, b, c\) so that \(\frac{a+1}{b}, \frac{b+1}{c}, \) and \(\frac{c+1}{a}\) are also integers.
6. For two non-zero real numbers $a, b$, the equation, $a(x - a)^2 + b(x - b)^2 = 0$ has a unique solution.
Prove that $a = \pm b$.

7. Consider a positive integer,
$$N = 9 + 99 + 999 + \ldots + 999\ldots9_{2018}$$
How many times does the digit 1 occur in its decimal representation?

8. A rectangular sheet of paper whose dimensions are 12 x 18 is folded along a diagonal, creating the M-shaped region drawn in the picture (see below).

Find the area of the shaded region.

9. Alice and Bob are playing the following game:
They take turns writing on the board natural numbers not exceeding 2018 (to write the number twice is forbidden).
Alice begins. A player wins if after his or her move there appear three numbers on the board which are in arithmetic progression.
Which player has a winning strategy?

10. There is a sequence of numbers +1 and -1 of length $n$. It is known that the sum of every 10 neighbouring numbers in the sequence is 0 and that the sum of every 12 neighbouring numbers in the sequence is not zero.
What is the maximal value of $n$?