ROBIN CONSTANTS ON ALGEBRAIC CURVES IN $\mathbb{C}^2$

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Central to the study of potential theory in $\mathbb{C}$ is the Green's function (with pole at infinity) for a compact set $K \subset \mathbb{C}$. Recall that the Green's function with pole at infinity, which we denote by g_K, is the unique function which has the following properties:

(i) $g_K : \mathbb{C}\backslash K \to (0, \infty)$ is harmonic;
(ii) $\lim_{z\to\infty}(g_K(z) - \log|z|$ exists and is finite (this value is known as the Robin constant);
(iii) for each $w \in \partial K$, $\lim_{z\to w} g_K(z) = 0$.

The importance of the Green's function stems from it's application in studying polar sets, polynomial approximation and growth properties of analytic functions (amongst other applications). In higher dimensions we construct functions with similar properties to that of the Green's function in one variable through a very different process, which we call pluricomplex Green's functions.

The pluripotential theory on an algebraic curve is closely related to that of $\mathbb{C}$. One notable result of this nature due to Sadullaev [2] is that the pluricomplex Green's function for an algebraic curve is harmonic (hence maximal) and for algebraic varieties this need not be true . This mirrors the fact that in $\mathbb{C}$ the Green's function is harmonic and that this need not be true for pluricomplex Green's functions in $\mathbb{C}^n, n \geq 2$. On the other hand, Ma'u [1] showed that an irreducible algebraic curve of degree d has d naturally arising Chebyshev constants with each Chebyshev constant corresponding to one of the asymptotes of the curve. This is in contrast to the one variable case where only one Chebyshev constant naturally arises for sets.

One aspect of our research was concerned with generalising the Robin constant to algebraic curves, which is closely related with the Chebyshev constants studied by Ma'u. We were able to show that when an algebraic curve of degree d is irreducible, there exists d Robin constants with each corresponding to an asymptote of the curve. Moreover we were able to recover a well known polynomial formulae concerning the Green's function and Robin constant from the one variable theory on algebraic curves. This culminated in a few results, the most notable being the equality of Robin constant and Chebyshev constant when they correspond to the same asymptote (as they do in the one variable case).

The main focus of this talk will be to illustrate how one variable potential theory (particularly that of Green's functions) can be used to establish the aforementioned results. To do this we will develop the necessary concepts in $\mathbb{C}$ (such as harmonic functions, subharmonic functions, Green's function and Robin constant) and progress into discussing the appropriate analogues in the algebraic curve case. We will then sketch the ideas behind a few arguments culminating in the existence of the directional Robin constant.

References

[1] S. Ma'u, *Chebyshev constants and transfinite diameter on algebraic curves in* $\mathbb{C}^2$, Indiana Univ. Math. J. **60** (2011), 1767-1796.

[2] A. Sadullaev, *An Estimate for Polynomials on Analytic Sets*, Math. USSR. Izvestiya, **20** (3) (1983), 493-502