

Positive Polynomials and Sums of Hermitian Squares

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Polynomials are the most basic functions in all of Mathematics. A well known theorem of Gauss states that any (univariate) non-negative polynomial over \mathbb{R} can be written as a sum of squares of other (univariate) polynomials [1].

A natural question that arises from this, is whether we can say anything similar about polynomials over \mathbb{C} . Specifically, we are interested in looking at trigonometric polynomials on the unit circle in \mathbb{C} . While the idea of ‘squares’ makes perfect sense in \mathbb{R} , when we switch to \mathbb{C} things are more subtle, and this is where we bring in Hermitian Squares.

In this talk I will be presenting a (non-standard) proof of the fact that any real, non-negative polynomial can be written as a sum of squares, with an argument based on the Hahn-Banach theorem, Convexity [2], and Hankel Matrices. I will also be introducing Hermitian Squares, along with their importance in mathematics.

References

- [1] Scott McCullough and Mihai Putinar. Noncommutative sums of squares. *Pacific J. Math*, 218(1):167–171, 2005.
- [2] Alexander Barvinok. *A course in convexity*, volume 54. American Mathematical Soc., 2002.