MAX: A Preview of Coming Attractions

Paddy and Peter, MAX Lecturers
Weeks 1-4: MATLAB and Cryptography

MAX UoA
Weeks 1-4: MATLAB and Cryptography

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Weeks 1-4: MATLAB and Cryptography

- MAX is a course that will teach you how to use MATLAB, a programming language + numerical computing environment.
- MATLAB can do some remarkably beautiful things!
- We’re not assuming that you know how to program entering this class; we’re going to build these skills from scratch.
- If you do know how to program, however, we’ll have plenty of things to keep you engaged :D
Weeks 1-4: MATLAB and Cryptography

Cryptography is the art of communicating securely... and also the art of breaking into those secure communications!

An example: the Caesar cipher.

Suppose your message is "i want tacos," and your secret key is 4.

Translate your message into numbers.

Then, add your key to each number!

i want tacos

↓

9 23 1 14 20 20 1 3 15 19

↓

13 1 5 18 24 24 5 7 19 23

↓

m aerx xegsw

a b c d e f g h i j k l m

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

n o p q r s t u v w x y z

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

14 15 16 17 18 19 20 21 22 23 24 25 26
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↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
```

```
n o p q r s t u v w x y z
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
↓
```

```
1 2 3 4 5 6 7 8 9 10 11 12 13
```

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  \[
  \begin{array}{c}
  9 \quad 23 \quad 1 \quad 14 \quad 20 \quad 20 \quad 1 \quad 3 \quad 15 \quad 19 \\
  \end{array}
  \]

  \[
  \begin{array}{c}
  13 \quad 1 \quad 5 \quad 18 \quad 24 \quad 24 \quad 5 \quad 7 \quad 19 \quad 23 \\
  \end{array}
  \]

  \[
  \begin{array}{c}
  m \quad a \quad e \quad r \quad x \quad x \quad e \quad g \quad w \\
  \end{array}
  \]
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i want tacos

  
  ↓

  
  i

  
  ↓

  
  9

  
  ↓

  
  23

  
  ↓

  
  1

  
  ↓

  
  14

  
  ↓

  
  20

  
  ↓

  
  20

  
  ↓

  
  1

  
  ↓

  
  3

  
  ↓

  
  15

  
  ↓

  
  19

  
  ↓

  
  m

  
  ↓

  
  a

  
  ↓

  
  e

  
  ↓

  
  r

  
  ↓

  
  x

  
  ↓

  
  e

  
  ↓

  
  g

  
  ↓

  
  s

  
  ↓

  
  w

  
  ↓

  
  a

  
  ↓

  
  b

  
  ↓

  
  c

  
  ↓

  
  d

  
  ↓

  
  e

  
  ↓

  
  f

  
  ↓

  
  g

  
  ↓

  
  h

  
  ↓

  
  i

  
  ↓

  
  j

  
  ↓

  
  k

  
  ↓

  
  l

  
  ↓

  
  m

  
  ↓

  
  n

  
  ↓

  
  o

  
  ↓

  
  p

  
  ↓

  
  q

  
  ↓

  
  r

  
  ↓

  
  s

  
  ↓

  
  t

  
  ↓

  
  u

  
  ↓

  
  v

  
  ↓

  
  w

  
  ↓

  
  x

  
  ↓

  
  y

  
  ↓

  
  z

  
  ↓

  
  14

  
  ↓

  
  15

  
  ↓

  
  16

  
  ↓

  
  17

  
  ↓

  
  18

  
  ↓

  
  19

  
  ↓

  
  20

  
  ↓

  
  21

  
  ↓

  
  22

  
  ↓

  
  23

  
  ↓

  
  24

  
  ↓

  
  25

  
  ↓

  
  26
Weeks 1-4: MATLAB and Cryptography

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\[
\begin{array}{cccccccccccccccccccccc}
9 & 23 & 1 & 14 & 20 & 20 & 1 & 3 & 15 & 19 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
13 & 1 & 5 & 18 & 24 & 24 & 5 & 7 & 19 & 23 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
& & m & a & e & r & x & x & & & e & & & g & s & w \\
\end{array}
\]
Weeks 1-4: MATLAB and Cryptography

- The Caesar cipher can be broken.
- But what about more sophisticated ciphers?

Are there encryption methods that are immune to attacks?
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```
 a  b  c  d  e  f  g  h  i  j  k  l  m  n  o  p  q  r  s  t  u  v  w  x  y  z
↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓  ↓
f  j  o  r  d  b  a  n  k  v  e  x  t  c  w  m  g  l  y  p  h  s  q  u  i  z
```

or
Weeks 1-4: MATLAB and Cryptography

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| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
| f | j | o | r | d | b | a | n | k | v | e | x | t | c | w | m | g | l | y | p | h | s | q | u | i | z |

or

jamisdelicious

+++ ++++++++ ++++++++ ++++++++ ++++++++ ++++
keykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykeykey

= tekswbopgmmmmw
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```
a b c d e f g h i j k l m n o p q r s t u v w x y z
```

or

```
jamisdelicious
```
```
+ + + + + + + + + + + + + +
```
```
keykeykeykeykeykeykeykey
```
```
teekswbopgmmmelew
```

- Are there encryption methods that are *immune* to attacks?
Weeks 5-8: Modelling, Fractals and Chaos

In the middle third of the course, you’re going to turn your MATLAB skills towards the field of mathematical modelling!

Here, you’ll study the equations used by the MoH in NZ to model COVID-19:

\[
\begin{align*}
S_{n+1} &= S_n - e \cdot t \cdot I_n \\
I_{n+1} &= I_n + e \cdot t \cdot I_n \cdot P - \gamma I_n \\
R_{n+1} &= R_n + \gamma I_n
\end{align*}
\]
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Weeks 5-8: Modelling, Fractals and Chaos

\[ x'(t) = \sigma (y(t) - x(t)) \]
\[ y'(t) = x(t) (\rho - z(t)) - y(t) \]
\[ z'(t) = x(t) y(t) - \beta z(t) \]

and the chaos game.
Weeks 5-8: Modelling, Fractals and Chaos

▶ You’ll also encounter **chaos**, in the **butterfly effect**
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MAX UoA
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Finally, you'll encounter fractals, and see how an outwardly simple function like $f_c(z) = z^2 + c$ can yield images like the following:
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Finally, you’ll encounter **fractals**, and see how an outwardly simple function like \( f_c(z) = z^2 + c \) can yield images like the following:

![Fractal Image]
Weeks 9-12: Graph Theory
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- The last third of this course is centered on graph theory!
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- With this tool, we’ll prove that there are only 5 platonic solids:
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Then, we'll prove that any map can be colored with at most four colors:
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To finish, we'll study random walk problems by using (of all things) circuits.
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