Next month Felix Baumgartner plans on breaking the world record for high altitude skydiving. He will make his jump from a capsule suspended beneath a balloon, at the edge of space. After Felix has landed, a remote triggering system will release the capsule from the balloon.

In the event that electronic tracking is unavailable, what size search area is required in order to retrieve the capsule?
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**Introduction:**
We begin by defining key terms and assumptions involved in the investigation.

The edge of space, defined to be the Karman line at 100 km\(^1\) was assumed to be an unfeasible height that could be reached. Instead, Felix’s mission is well documented online, where it is planned that he will descend from a height of 120 000 feet, converted to approximately 37 000 m. This will be the definition used for the ‘Edge of Space’ throughout this investigation.

In order to answer the question it was necessary to evaluate several aspects. Included are the extent to which air density affects the drag force of Mr Baumgartner and the capsule, their relative dimensions and potential wind speeds. These have been separated into their respective atmospheric regions; the Stratosphere and the Troposphere.

It is assumed that the searchers specified in the investigation topic can identify where Felix lands and where the capsule was released.

Determining Dimensions:

A preliminary complication identified in this report was the respective dimensions and orientations in free fall of the different projectiles – Felix, and the Capsule. It was assumed that Felix was to be falling vertically headfirst from the delta position\(^2\) with dimensions of 0.6m across the shoulders and 0.4m from front to back.

Using this assumption, it was possible to treat the vertical and horizontal components of velocity independently. In addition, research has found that skydivers have a drag coefficient\(^3\) of 0.4 – 1 and it was concluded that with the correct aerodynamic apparel, Felix would have a drag co-efficient of 0.4 in vertical motion. It was also found that Felix would have some horizontal velocity due to forces exerted by the wind. Therefore, his drag profile in the horizontal motion would best be represented by a flat plate travelling against air flow which is a value of 1.28\(^4\). His mass was found to be 260 pounds\(^5\), which is then converted to 118 kg.

The capsule resembled a cylinder and the official Red Bull website revealed dimensions of a radius of 1.2m and height of 3.4m. It was assumed that the capsule falls vertically downwards with the base parallel to the surface of the earth on the bottom. This resulted in the capsule being treated as a circular drag profile in vertical motion with a drag co-efficient of 1.14. In horizontal motion it was determined that the value was 1.17\(^6\). It has a mass of 1315kg.

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Average Air Densities

From the altitude (h) to density (ρ) graph on the left, a model was developed to determine the average density for the troposphere and the stratosphere.

The following equation was modelled with coordinate mapping techniques.

\[ h = 38e^{-2.695\rho} - 1 \]

As evident from the graph, the lower limit of the stratosphere is \( h = 9 \) km.

\[ \Rightarrow 9 = 38e^{-2.695\rho} - 1 \]
\[ \Rightarrow \rho = 0.495 \text{ kg m}^{-3} \]

The area under the curve was then found from these limits, that is, \( \rho = 0 \) to \( \rho = 0.495 \).

\[ A = \int_{0}^{0.495} (38e^{-2.695\rho} - 1) \, d\rho \]
\[ A = \left[ -14.1002e^{-2.695\rho} - \rho \right]_{0}^{0.495} \]
\[ A = \left[ (-3.714 - 0.495) + 14.1002 \right] \]
\[ A = 9.891 \]

We now suppose that this area is a rectangle with the length as the density and the width (w) as the altitude. Dividing by the altitude, we obtain an average density value for the stratosphere.

\[ \rho_{Average} = \frac{A}{w} = \frac{9.891}{37} = 0.2673 \text{ kgm}^{-3} \]

Similarly, for the troposphere, the lower limit is found when \( h = 0 \).

\[ \Rightarrow 38e^{-2.695\rho} = 1 \]
\[ \Rightarrow \rho = 1.3498 \]

The area under the curve is then found using the limits \( \rho = 0.495 \) and \( \rho = 1.3498 \).

\[ A = \int_{0.495}^{1.3498} (38e^{-2.695\rho} - 1) \, d\rho \]
\[ A = \left[ -14.1002e^{-2.695\rho} - \rho \right]_{0.495}^{1.3498} \]
\[ A = 2.029 \]
Dividing by the altitude, and adding on the rectangle’s average density of 0.495.

\[ \rho_{\text{Average}} = \frac{A}{w} = \frac{2.029}{9} + 0.495 = 0.7204 \]

**Evaluating Wind Speeds**

Wind speeds were explored using the maximum potential wind velocity allowed for launch. This allowed for the search area calculated to represent the worst-case scenario and gave no possibility of the capsule’s landing point exceeding this region.

From research, a model for the wind speed was discovered.

From this graph the average wind speed has been approximated by numerical integration methods with respect to the y-axis, finding the area, and then dividing by the length to account for an average wind speed for the stratosphere and troposphere. The section from 30 to 12 km has been integrated using a mixture of Numerical methods, due to its polynomial appearance and its odd number of intervals. Each subsequent value for the wind speeds at each interval has been approximated according to the graph.

The Simpson’s Rule states that:

\[ \int_{a}^{b} f(x) \, dx \approx \frac{1}{3} h[y_0 + y_n + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})] \]

Where \( h = \frac{b-a}{n} \), \( y_r = f(x_r) \) and \( n \) is even.

\[ \Rightarrow \int_{14}^{30} f(x) \, dx \approx \frac{1}{3} \times \frac{30 - 14}{8}[28 + 48 + 4(24 + 24 + 30 + 42) + 2(22 + 27 + 33)] \]

\[ \Rightarrow \int_{a}^{b} f(x) \, dx \approx 480 \text{units}^2 \]

Since \( n \) must be even, the last section of the stratosphere is calculated as a trapezium due to its relatively straight nature.

The wind speed at 12 km is approximated to be 65 m/s. Hence, the area of the section from 14 to 12 km is calculated to be

\[ A = \frac{(48 + 65) \times 2}{2} = 113 \text{ units}^2 \]

The section from 12 to 0 km has been integrated by the Trapezium Rule as it tends towards a linear graph.

The Trapezium Rule states that:

\[ \int_{a}^{b} f(x) \, dx \approx \frac{1}{2} h[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})] \]
Where \( h = \frac{b-a}{n}, y_r = f(x_r) \)
\[
\Rightarrow \int_0^{12} f(x) \, dx \approx \frac{1}{2} \times \frac{12 - 0}{6} \left[ 6 + 90 + 2(13 + 23 + 40 + 52 + 70) \right]
\]
\[
\Rightarrow \int_a^b f(x) \, dx \approx 492 \text{ units}^2
\]
Hence the average wind speed of the stratosphere is approximated to be
\[
v = \frac{480 + 113}{18} = 32.94 \text{ ms}^{-1}
\]
And for the troposphere,
\[
v = \frac{492}{12} = 41 \text{ ms}^{-1}
\]

**Vertical Terminal Velocity Calculations**

The terminal velocity, the velocity at which gravity and air drag are equal, for the stratosphere can be calculated with the following equation:

\[
V_T = \sqrt{\frac{2mg}{\rho AC_d}}
\]

- \( V_T \) = Terminal velocity
- \( m \) = Mass of falling object
- \( g \) = Acceleration due to gravity
- \( \rho \) = Mass Density of air
- \( C_d \) = Drag Co-efficient
- \( A \) = Cross-Sectional area of the object

**Felix:**

**Stratosphere:**
\( m \) (Felix) = 118, \( g = 9.81 \), \( \rho = 0.2673, C_d = 0.4 \)
The cross-sectional area of Felix in vertical motion is calculated using that of an ellipse as this is the most closely related geometrical shape to the falling headfirst body of Felix.

\( A = \pi \times 0.3 \times 0.2 = 0.1885 \text{ m}^2 \),
Hence,
\[
V_T = \sqrt{\frac{2 \times 118 \times 9.81}{0.2673 \times 0.1885 \times 0.4}} = 338.93 \text{ ms}^{-1}
\]
By the mechanics equations, the displacement of Felix and the time taken for terminal velocity to be reached can be found:
\[
s = \frac{v^2 - u^2}{2a} = \frac{338.93^2 - 0^2}{2 \times 9.81} = 5854.795 \text{m}
\]
And
\[
\frac{v = u + at}{v - u} \Rightarrow t = \frac{a}{a} = 34.549 \text{s}
\]
This will be classified as the first part of the journey.

The second part of the journey is made up of the rest of the time spent in the stratosphere.

\[ t = \frac{d}{v} = \frac{28000 \text{ (thickness of the stratosphere)} - 5854.795}{338.93} = 65.339 \text{ s} \]

Therefore, 65.339s is spent in the second part of the journey.

**Troposphere**

Due to different air densities in the different atmospheric regions the terminal velocity has changed:

\[ V_T = \sqrt{\frac{2mg}{\rho AC_d}} = \sqrt{\frac{2 \times 118 \times 9.81}{0.7204 \times 0.1885 \times 0.4}} = 206.451 \text{ms}^{-1} \]

Because Felix is travelling at a velocity that is greater than the terminal velocity in the Troposphere, he must decelerate. The unbalanced force of air resistance provides this deceleration. Because the frictional force is proportional to Felix’s velocity, and acceleration is the derivative of velocity, a differential equation can be used to model the deceleration:

\[ m \frac{dv}{dt} = -mg - kv \]
\[ 118 \frac{dv}{dt} = -118 \times 9.81 - 0.027156v \]
\[ \Rightarrow \frac{dv}{dt} + 0.00023014v = -9.81 \]

We use the integrating factor method to solve the differential equation, where the integrating factor is

\[ \mu(t) = e^{\int 0.00023014 \, dt} = e^{0.00023014t} \]

Therefore,

\[ v = \frac{1}{\mu(t)} \left( \int \mu(t) \times (-9.81) \, dt \right) \]
\[ e^{0.00023014t} v = \int -9.81 \times e^{0.00023014t} \, dt \]
\[ e^{0.00023014t} v = -9.81 \times \left[ \frac{e^{0.00023014t}}{0.00023014} \right] + C \]
\[ \Rightarrow v = \frac{-9.81}{0.00023014} + \frac{C}{e^{0.00023014t}} \]

When \( t = 0, \ v = 338.93 \)
\[ \Rightarrow C = 42626.228 \]
\[ \Rightarrow v = -42626.22 + \frac{42626.228}{e^{0.00023014t}} \]

When \( v = 206.451, \ e^{0.00023014t} = 0.99518 \)
\[ \Rightarrow t = 20.9941 \]
\[ \Rightarrow a = -6.31 \text{ ms}^{-2} \]
Therefore the distance travelled during section 3, when Felix is decelerating in the troposphere, is displaced 5725.161m downwards over a time of 20.9941s.

The fourth section refers to after the terminal velocity has been reached:

\[ s = vt \]

\[ s = 206.451 \times 8.4807 = 1750.839\text{m} \]

Section 5 occurs at 5000 feet (1524m) when Felix opens his parachute.

\[ t = \frac{s}{v} \]
\[ v = 2.235 \text{ms}^{-1} \]

(A safe landing velocity was regarded as such)

Therefore,

\[ t = \frac{1524}{2.235} = 681.879\text{s} \]

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<th>Section</th>
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<th>Distance (m)</th>
</tr>
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</tr>
<tr>
<td>Total</td>
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</tr>
</tbody>
</table>

Capsule:

Stratosphere:

\[ V_T = \sqrt{\frac{2mg}{\rho AC_d}} \]

\[ V_T = \sqrt{\frac{2 \times 1315 \times 9.81}{0.2673 \times \pi \times 1.2^2 \times 1.14}} \]

\[ V_T = 136.806 \text{ms}^{-1} \]

\[ t = \frac{v - u}{a} \]

\[ s = ut + \frac{1}{2}at^2 \]

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<th>Section</th>
<th>t</th>
<th>s</th>
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<td>952.599</td>
</tr>
<tr>
<td>2</td>
<td>212.325</td>
<td>29047.401</td>
</tr>
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</table>

Section 1 models the capsule’s motion as it is released from the balloon with an upwards velocity of 2.5 ms\(^{-1}\) from a point 2.5 \times 811 \approx 2000 \text{m} above where Felix was released from (altitude of 39 000 m).
Section 2 is defined in the same manner as previously for Felix.

**Troposphere**

\[ V_T = \sqrt{\frac{2 \times 1315 \times 9.81}{0.7204 \times \pi \times 1.2^2 \times 1.14}} \]

\[ V_T = 83.332 \text{ m} \]

Assuming that the capsule decelerates almost instantaneously due to the high drag coefficient, it travels through the troposphere at 83.332 ms\(^{-1}\). The distance travelled through the troposphere without a parachute (assuming the parachute is deployed at the same altitude as Felix’s parachute) is given by:

\[ d = 9000 - 1524 = 7476 \text{ m} \]

Therefore, the time taken is equal to:

\[ t = \frac{d}{v} \]

\[ t = \frac{7476}{83.332} = 89.713 \text{ secs} \]

Assuming that the parachute is deployed at an altitude of 1524 m and with a velocity equal to Felix’s when the parachute is opened (of 2.235m/s), the capsule will travel through the troposphere with the parachute deployed for a time of:

\[ t = \frac{1524}{2.235} = 681.879 \text{ secs} \]

**Horizontal Displacement Calculations**

Due to wind force horizontal displacement will occur. The equation for wind-force i.e. the drag equation, is as follows:

\[ F_D = \frac{1}{2} \rho v^2 C_d A \]

The values are substituted below to find the respective accelerations for both the stratosphere and troposphere:

**Felix**

**Stratosphere**

\[ F_D = \frac{1}{2} \times 0.2673 \times 32.94^2 \times 1.28 \times 1.08 = 200.47\text{N} \]

By \( F_D = ma \), where \( m = 118\text{kg}, \alpha = 1.699 \text{ ms}^{-2} \)

However, assuming he is already travelling at the stratospheric wind speed his velocity will remain constant at 32.94 ms\(^{-1}\)

\[ ^7\text{Wikipedia: } http://en.wikipedia.org/wiki/Drag_equation \]
As calculated prior, Felix takes 99.889s to move from the stratosphere to the troposphere.

**Section 1:**

\[ d = vt \]
\[ d = 32.94 \times 99.889 \]
\[ d = 3290 \text{ m} \]

**Troposphere**

Because the wind speed is greater in the troposphere Felix will accelerate to this speed (41 ms\(^{-1}\))

\[ F_D = \frac{1}{2} \times 0.7204 \times 41^2 \times 1.28 \times 1.08 \]
\[ F_D = 837.04 \text{ N} \]

By \( F_D = ma \),

Where \( m = 118 \text{ kg} \),
\[ a = 7.0935 \text{ ms}^{-2} \]

Therefore, in the troposphere there are three sections, one where the velocity is increasing until it reaches horizontal equilibrium, one at horizontal equilibrium and the third after the parachute is released.

**Section 2:**

\[ t = \frac{v-u}{a} = \frac{41-32.94}{7.0935} \]
\[ t = 1.136 \text{ s} \]
\[ S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 7.0935 \times 1.136^2 \]
\[ S = 4.579 \text{ m} \]

**Section 3:**

From vertical calculations:

\[ t = (20.9941 + 8.4807) - 1.136 \]
\[ t = 28.33855 \text{ s} \]
\[ S = vt = 28.33855 \times 41 \]
\[ S = 1161.8785 \text{ m} \]

**Section 4:**

\[ S = vt \]
\[ S = 2.682 \times 681.879 = 1828.8 \text{ m} \]

Adding together sections 1-4 gives a total horizontal displacement of 6285m

**The Capsule**

While in the stratosphere, the capsule continues to travel in the direction of the wind at a speed of 32.94 m/s. Before Felix lands and it is released from the balloon, the capsule will travel a distance of:

\[ d = vt \]
After the capsule is released, it is in the stratosphere for a time of:

\[ t = 14.4634 + 212.325 = 226.7884 \]

Which means that it travels horizontally through the stratosphere for a distance of:

\[ d = vt \]
\[ d = 32.94 \times 226.7884 = 7470.4099m \]

When the capsule moves into the troposphere, the increased wind speeds exert a force on the capsule:

\[ F = \frac{1}{2} \rho v^2 C_m A \]
\[ F = \frac{1}{2} \times 0.7204 \times (41)^2 \times 1.17 \times 8.16 = 5780.793 \]

Which gives an acceleration:

\[ a = \frac{F}{m} = \frac{5780.793}{1315} = 4.396m/s^2 \]

Therefore, the capsule accelerates to the wind speed of 41 m/s in the troposphere in a time of:

\[ t = \frac{v - u}{a} = \frac{41 - 32.94}{4.396} = 1.833 \text{ secs} \]

In which time it travels:

\[ s = ut + \frac{1}{2} at^2 = (32.94 \times 1.833) + \left(\frac{1}{2} \times 4.396 \times 1.833^2\right) = 67.783m \]

After it reaches the wind speed, which acts as the terminal horizontal velocity, the capsule travels through the troposphere for an additional time of 87.88 seconds. Therefore, it travels a horizontal distance of:

\[ d = vt = 41 \times 87.88 = 3603.08m \]

After the parachute is deployed, the parachute decelerates almost instantly due to its high drag coefficient. Assuming it travels at the wind speed of 2.682 m/s, and travels with the parachute deployed for 681.879 seconds, then it will travel an additional horizontal distance of:

\[ d = vt = 2.682 \times 681.879 = 1828.7995m \]

Therefore, the total horizontal distance travelled by the capsule from the point where Felix is released is:

\[ d = 26722.305 + 7470.410 + 67.783 + 3603.08 + 1829 = 39692m \approx 40000m \text{ (1 s.f.)} \]
Conclusion

The first diagram above shows a simplified path of Felix and the capsule during the stunt. At point P the balloon is launched and the balloon rises until point J where Felix jumps from the capsule and lands at point F. The capsule continues to rise and move with horizontal velocity until point Q where it is released from the balloon. From point Q the capsule can travel in any
direction depending on whether there is a change in the wind direction after Felix lands. \( C_1 \) is the point at which the capsule will return to Earth assuming there is no change in direction of the wind and point \( C_2 \) is the point at which the capsule will return to Earth assuming the wind experiences a 180 degree change in direction and remains at the same speed. Point \( O \) is the centre of the circle in which the capsule will possibly land with radius \( R \) assuming wind speed remains constant. However in determining a search area from the capsule the radius of the search area, in order to be sure of acquiring the capsule should be the maximum displacement the capsule can reach from Felix. This is radius \( L_{\text{max}} \). From this it can be concluded that the search radius should be centred 40 km around Felix, giving a search area of \( 5027 \text{ km}^2 \).

**Evaluation of Assumptions and Limitations:**

There are many limitations and assumptions made in the evaluation of this problem.

**Acceleration of Gravity:** We assumed that the acceleration due to gravity was constant throughout free fall. In reality this would not be the case however the difference in acceleration due to gravity at the different altitudes is relatively insignificant in the calculations. To make the calculations more accurate the change in the Earth’s gravitational potential as altitude decrease could have been incorporated into equations.

**Drag coefficients:** Another assumption made was the drag coefficient of Felix’s body was 0.40 during free fall by assuming that he was facing vertically down as he fell. This gives an indication of his real drag coefficient however will not be entirely accurate. We also assumed that his drag coefficient due to the horizontal force of the wind was 1.28; that of a flat plate. This is also not entirely accurate however it approximates the general shape of the space suit. For the capsule with a conical shape the drag coefficient during free fall was 1.14, and the rounded surface of the capsule gives it a drag coefficient of 1.17 from the wind.

**Weather:** From research we found that they would determine it unsafe to launch if the wind speed on the ground was 6 mph or more. Therefore we concluded it safe to assume that this would be the maximum speed of the wind at the Earth’s surface while the jump was performed. In reality it is possible that the weather could unexpectedly change however this is not able to be predicted in calculations. They also would only launch if the skies were clear with no clouds and visibility was 4.8 km so we assumed that the air density would not be influenced by clouds.

**Radius of Search Area:** When interpreting the question we assumed that the radius of the search area is centred on the point from which Felix lands not from the Roswell Space Centre. Assuming this, the search radius is the maximum possible displacement between Felix and the space capsule from the time that he leaves the capsule until the time that the capsule lands back on Earth. This is to make sure there is a margin of uncertainty and ensure that the capsule is within the search area.

**Air Density and Wind Speeds:** As Felix and the space capsule return to Earth, air density is increasing constantly. However we divided the descent of Felix into 5 sections, when he
accelerates from the capsule, when he’s at terminal velocity in stratosphere, decelerates in troposphere, terminal velocity in troposphere and then parachuting. We divided the descent of the capsule into 4 sections, accelerating after release, terminal velocity in stratosphere, terminal velocity in troposphere and then parachuting. Due to the much higher drag coefficient of the capsule, we assumed that the deceleration would be much faster between the layers and so would be negligible. This is somewhat arbitrary; while the deceleration will be much greater it would be better to still incorporate into the calculations. For each of the sections different values of air density and wind speeds were used based on calculations of averages for those sections. While this misrepresents the true values somewhat it gives a solid approximation of the effects of the wind and air densities in these sections. We also assumed that the only possible changes in the direction of the wind would be as Felix landed and that the speeds of the wind in each section remained the same despite any change in direction. This is not the case as winds are subject to be in different directions at different altitudes however since we are researching the maximum possible displacement due to the wind it was required that the wind speed remained constant in direction throughout the ascent and descent of the capsule.

Ascent of Balloon after jump: In the time that it takes for Felix to descend to the ground the balloon continues to rise and will also have a horizontal velocity due to the force it received from the wind while moving through the stratosphere. We assumed that the rate of ascent would be approximately the average rate of ascent of 5.08 m.s\(^{-1}\) and that the horizontal velocity after he jumps is the same as the horizontal velocity in the stratosphere; 32.94 m.s\(^{-1}\). This is likely to be an imperfect measure as the effect of decreased air pressure and higher altitudes on the lift force given by a helium balloon were not investigated in this report and are unlikely to remain constant.

Jet streams: Jet streams are areas of high wind speed in the area near the boundary between the stratosphere and the troposphere. They would have a significant influence on the horizontal displacement of Felix however from the choice of location of the stunt they are in an area with minimal influence from jet streams and that is unlikely to change during the stunt (Wikipedia, 2012).

Rotation of the Earth: During the time that Felix and the capsule descend, the Earth will rotate by some margin. Due to the different displacements of Felix and the capsule relative to the Earth this may contribute to the overall horizontal displacement between Felix and the capsule, however in comparison to the effects due to the wind this is unlikely to be as significant. Although this is somewhat arbitrary it would be different for each wind direction and would influence the magnitude of the search area.
Bibliography


