Modeling solid Fracture using Peridynamics

Hooman Zarreh, Biswajit Banerjee, Florin Bobaru, and Kumar Mithraratne

Auckland Bioengineering Institute, The University of Auckland, Auckland, NZ

Introduction

Fracture mechanics is the field of mechanics concerned with study of the propagation of cracks in materials.

The prediction of initiation and propagation of multiple cracks, has a number of applications such as those in visual effect industry as well as some medical applications like bone fracture.

The main goal of the research program is to develop an integrated computational framework for simulating fracture of solid bodies efficiently and accurately.

Since closed form analytical solutions do not generally exist, several numerical methods are used to find approximate solutions for rigid body motion, deformations and ensuing fracture.

- **Finite element method (FEM)** is one the most widespread numerical techniques that discretizes spatial derivatives to calculate the approximate solutions of Partial Differential Equations (PDE). However, this method suffers from mesh distortion when the body undergoes large deformation resulting in numerical issues.
- **Material Point Method (MPM)** [4] is an Arbitrary Lagrangian Eulerian method that utilizes moving material particles and fixed background Eulerian mesh. Since MPM combines the best features of both Lagrangain and Eulerian descriptions, the method can deal with large deformation. On the other hand, local continuum mechanics models such as MPM and FEM, use PDE with spatial derivatives which do not exist in spatial discontinuities like the crack tip or surface. The local methods, thus, need more geometrical information of initial crack and they are not able to predict the initiation of failure. However, nonlocal continuum mechanics models [1], usually exploit integro-differential equations instead of partial differential equations. Therefore, these models are naturally able to handle spatial discontinuities and predict crack propagation better than the local models.
- **Peridynamics** [2] is a nonlocal mesh-free continuum method which does not use stress and strain tensors. Thus, the governing equation of the model and constitutive properties of materials do not have any spatial derivatives. Although Peridynamics is more suitable to predict the static and dynamic behavior of materials fracture, the method is relatively expensive in comparison with local methods like FEM.

Integrated Computational Framework

In order to have an accurate and computationally efficient method, an integrated computational framework will be developed incorporating Finite Element Method (FEM), Material Point Method (MPM), and Peridynamics to simulate deformations with no mesh distortion (positive jacobian), large deformations, and fracture of solid bodies respectively.

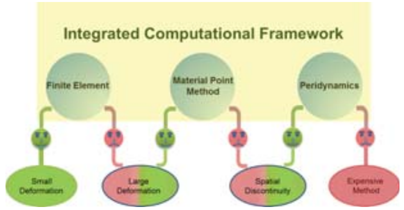


Fig. 1 Advantages and disadvantages of FEM, MPM, and Peridynamics

If the solid body has an initial crack, the Integrated Computational Framework (ICF) uses Peridynamics to predict crack propagation. Otherwise, Finite Element Method is used to simulate small deformation until a crack (or cracks) is (are) created.

If FEM detects that the body encounters large deformation with mesh distortion, Material Point Method takes over to simulate the large deformation until fracture events (e.g. crack initiation) are imminent.

ICF then switches to Peridynamics to handle crack propagation and fracture.

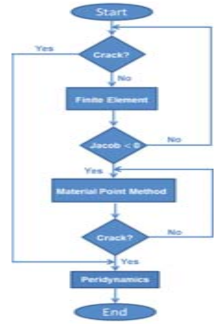


Fig. 2 Flowchart of Integrated Computational Framework

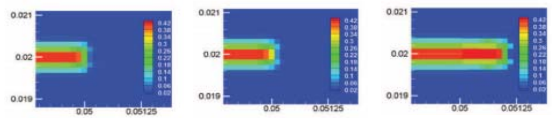
Peridynamics

There are two different approaches, bond-based and state-based, used in Peridynamics. The bond-based Peridynamics governing equation is a relation between the acceleration of a material point $\mathbf{x}$, density ρ, and body force $\mathbf{b}(\mathbf{x},t)$:

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathcal{H}_x} \mathbf{f}\left(\mathbf{u}(\hat{\mathbf{x}},t) - \mathbf{u}(\mathbf{x},t),\, \hat{\mathbf{x}} - \mathbf{x}\right) dV_{\hat{\mathbf{x}}} + \mathbf{b}(\mathbf{x},t).$$

The point $\hat{\mathbf{x}}$, which is a material point in $\mathcal{H}_x$ called the horizon of $\mathbf{x}$ (usually a sphere with center $\mathbf{x}$ and radius δ in 3D), exerts the pairwise force function $\mathbf{f}$ on $\mathbf{x}$. The pairwise force $\mathbf{f}$ depends on the bond $\boldsymbol{\xi} = \hat{\mathbf{x}} - \mathbf{x}$ (the relative position in reference configuration) and the relative current displacement $\boldsymbol{\eta} = \mathbf{u}(\hat{\mathbf{x}},t) - \mathbf{u}(\mathbf{x},t)$. It also contains the constitutive properties of the material.

Bobaru, et al. [3] for instance, applied the bond-based Peridynamics to model the 2D dynamic crack propagation in the brittle material Duran 50 glass with an initial crack. 12 MPa traction force has been applied along the upper and lower edges of the sample. The figure below shows the crack tip view after $3\mu s$, $6\mu s$, and $9\mu s$.



$3\mu s$ $6\mu s$ $9\mu s$

Fig. 3 Crack propagation- after $3\mu s$, $6\mu s$, and $9\mu s$

Bond-based Peridynamics uses central potential force. It means that the force between two points is a function of the distance between the points and acts in the line joining the points. Cauchy's relation for a lattice points system shows that the Poisson's ratio should be 0.25 when the points interact through the central force. For instance, a preliminary investigation shows that in a three dimensional cubic lattice with 8 points and 28 linear springs (one spring between any pair of points) if the points of the lower surface are fixed and the points of the upper surface are applied by a constant prescribed force (along the z-axis), then the upper points oscillate permanently and the average of Poisson's ratio is 0.2548 (during 250000 iterations).

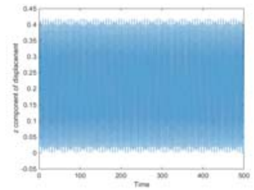


Fig. 4 Oscillation of the system

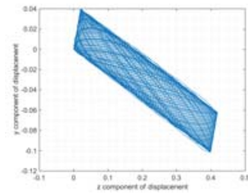


Fig. 5 Relation between the components of displacement with slope -0.25

Therefore, the bond-based Peridynamics models are restricted to a Poisson's ratio 0.25. Consequently, a new approach called state-based Peridynamics [5] was proposed in which the force of each bond depends on collective deformation of all bonds within the horizon of each point. This method, hence, can model solid materials with any Poisson's ratio.

References

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Future Work

- Implement 3D constitutive model of an orthotropic material (e.g. wood) in the model.
- Develop a stand-alone Material Point Method (MPM) code with constitutive law of an orthotropic material.
- Integrate FEM, MPM, and Peridynamics to simulate deformation and fracture.
- Validate the results from the model using the experimental data obtained under controlled conditions.

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