# An Efficient Computational Model for Respiratory Gas Exchange

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## Motivation

### Problem

- A computational model that can predict respiratory system response to pathology has many clinical applications, but current models are either too simplistic to represent structural heterogeneity, or too complex to solve rapidly.

### Aims

- To develop an efficient model to predict patient response in 'real-time' for clinical application.
- Require structural & functional detail to describe heterogeneity & matching of ventilation (V) & perfusion (Q) and nonlinear binding of $O_2$ to haemoglobin (Hb). To investigate to what extent structure & biophysical equations are required to capture spatial & temporal gas exchange function.

## Full Model

- A 3D anatomically structured element model of the human airways [1] coupled to 32000 acini at terminal branches (Fig 1).

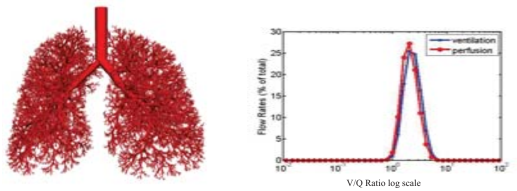


**Figure 1:** A) 3D finite element model of human lung airway B) Input V & Q distribution for normal conditions

- Finite deformation elasticity for soft tissue coupled to airways resistance & tissue compliance for V distribution [2]
- Multi-scale model for Q distribution [3].
- Advection-Diffusion Equation (ADX) and gas exchange (Eqn1) is solved for every acinus at each time step for multiple breath till equilibrium.

$$(1) \quad \frac{dPbO2}{dt} = \frac{Tt_{O2}}{\sigma_{O2} V_b}\left(1 + \frac{4[Hb]_b F'}{\sigma_{O2}}\right)^{-1}(PA_{O2} - Pb_{O2})$$

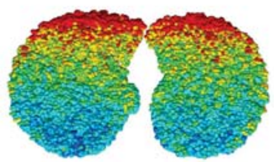

**Figure 2:** Full model predicted $PaO_2$ distribution of supine lung (anterior-posterior direction). Red: 120mmHg Blue: 80mHg

- The Model predicts physiologically consistent distribution of $P_aO_2$ for normal conditions (Fig 2).

## Gas Exchange Simplification

- We investigated the effect that an empirical equation (Eqn 2) fitted to $P_bO_2$ profile (Fig 3) instead of using gas exchange equation (Eqn 1) has on alveolar-arterial oxygen partial pressure $P(A\text{-}a)O_2$ drift.

$$(2) \quad P(t) = a + \frac{b}{exp^{ct}}$$

a- systemic venous $PO_2$
b- equilibrated $PcO_2$
c- parameterized by transit time

- As seen in Fig3, empirical equation can model $P_bO_2$ profile and save computational time by nearly 80% (Table1) .

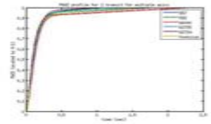

**Figure 3:** Characteristic profile of $P_bO_2$ for acini

## Geometric Simplification

- We investigated the impact of grouping increasing numbers of acini together to create a simplified geometry.
- Neighbouring acini are successively grouped together as lumped units, with their V & Q averaged.
- Asymmetric conducting airways leading to acini are replaced with symmetric airways, but anatomical dead space kept constant.
- Sequential averaging of acini reduces $P(A\text{-}a)O_2$ as seen in Fig 4.
- Sharp decrease at Horsfield order 7, below which averaging had 'minimal' effect on $P(A\text{-}a)O_2$. Optimal unit of V is ~7, consistent with literature [4].

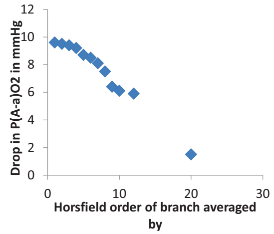


**Figure 4:** $P(A\text{-}a)O_2$ predicted for averaging acini of higher Horsfield order successively

## Numerical Methods

- Lagrange-Galerkin method (LGM) with split operator approach used to solve ADX equation.
- Functional Separation of geometric structure, diffusion only solved for Peclet number<2.
- Adaptive time stepping using predictor-corrector scheme implemented, $2^{nd}$ order Adam-Bashford as predictor

## Model Predictions

- Modified model is able to capture a large extent of spatial information in $PaO_2$ distribution (Fig 5). Gradient is preserved, though slice mean is slightly higher & variance is smaller.
- Modified model compare well with full model at ~2mmHg $P(A\text{-}a)O_2$ , at ~60% less computation time

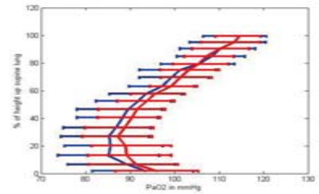

**Figure 5:** $PaO_2$ distribution up supine lung in 12mm iso-gravitational slice. Blue: full model Red: modified

| | $PAO_2$ | $PaO_2$ | $P(A\text{-}a)O_2$ | ADX time | GX time | Total |
|---|---|---|---|---|---|---|
| Full | 103.2 | 93.6 | 9.6 | 3hr05min | 2hr18min | 5hr23min |
| Modified | 103.2 | 95.7 | 7.5 | 1hr01min | 35min | 1hr36min |

**Table 1:** Prediction and efficiency comparison of models

## Comparison with MIGET

- We simulated 'virtual MIGET' using our model, to compare with MIGET experiments.

Prescribe V&Q distribution for normal[5], emphysema [6] lung condition according to experiment

Simulate inert gas exchange for 6 gases to obtain excretion/retention data

Perform MIGET analysis using excretion data to obtain V & Q for experimental comparison (analysis coded according to [5], in MATLAB)

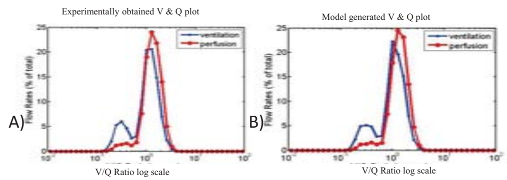


**Figure6:** Comparison of prescribed & model derived MIGET plot for V&Q for emphysema lung A)experiment B) model generated

- Comparison of prescribed V & Q distribution with virtual MIGET derived V/Q plot show good agreement for emphysema affected lung

## Conclusion

- We have developed an efficient model that gives good prediction of normal lung and a diseased lung condition.
- Trade off between loss of information against computational expense must be considered for specific applications.
- A functionally determined 'Optimal unit of ventilation is ~7 units.

### References

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